Emulation of the cosmic density field

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Image source: NASA/Apollo 17 crew; taken by either Harrison Schmitt or Ron Evans - https://web.archive.org/web/20160112123725/http://grin.hq.nasa.gov/ABSTRACTS/GPN-2000-001138.html (image link);

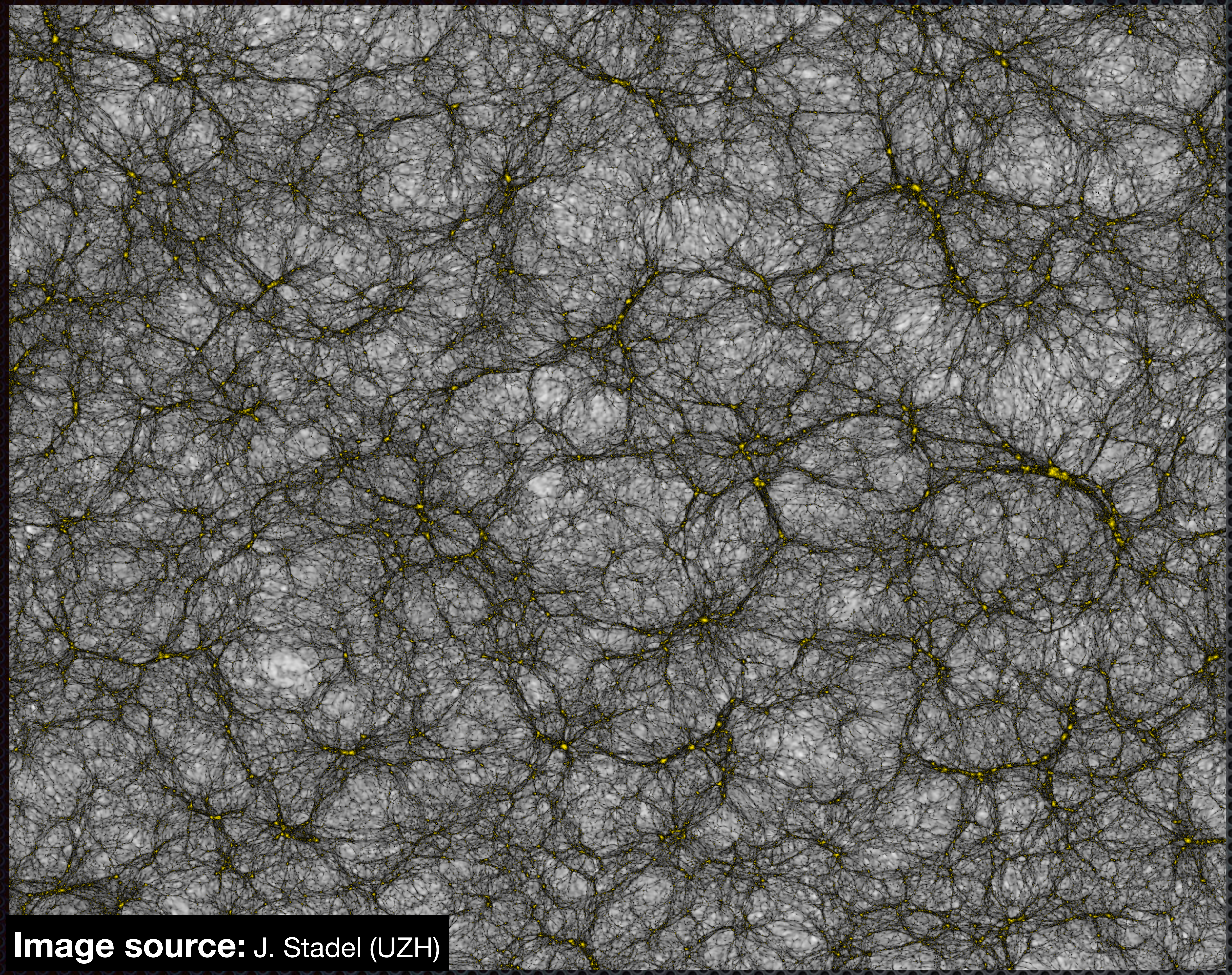
Image source: https://wtamu.edu/~cbaird/sq/2013/08/28/when-do-the-planets-in-our-solar-system-all-line-up/

Image source: https://www.starobserver.org/tag/andromedagalaxie/page/2/; **Credits:** Robert Gendler

Image source: Doug Potter (UZH)

Image source: J. Stadel (UZH)

The cosmic density field



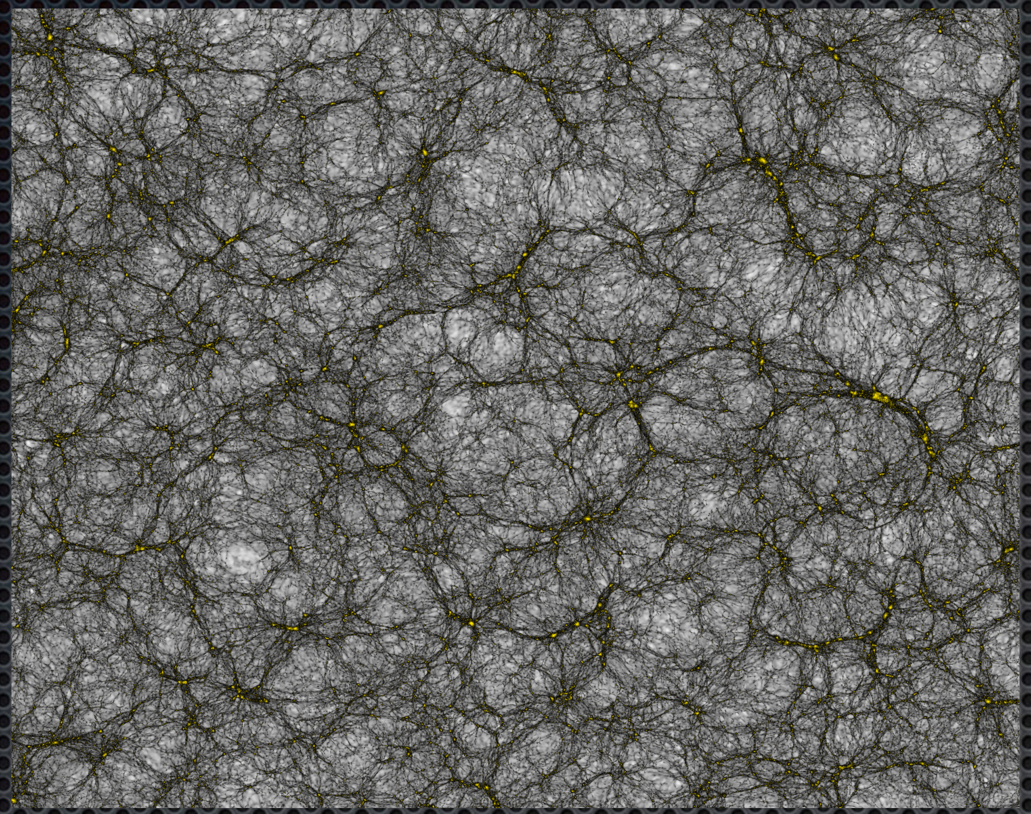
is described by fundamental properties of the Universe (called „cosmological parameters“) such as

- the amount of matter
- the amount of radiation
- amplitude of initial density fluctuations
- others

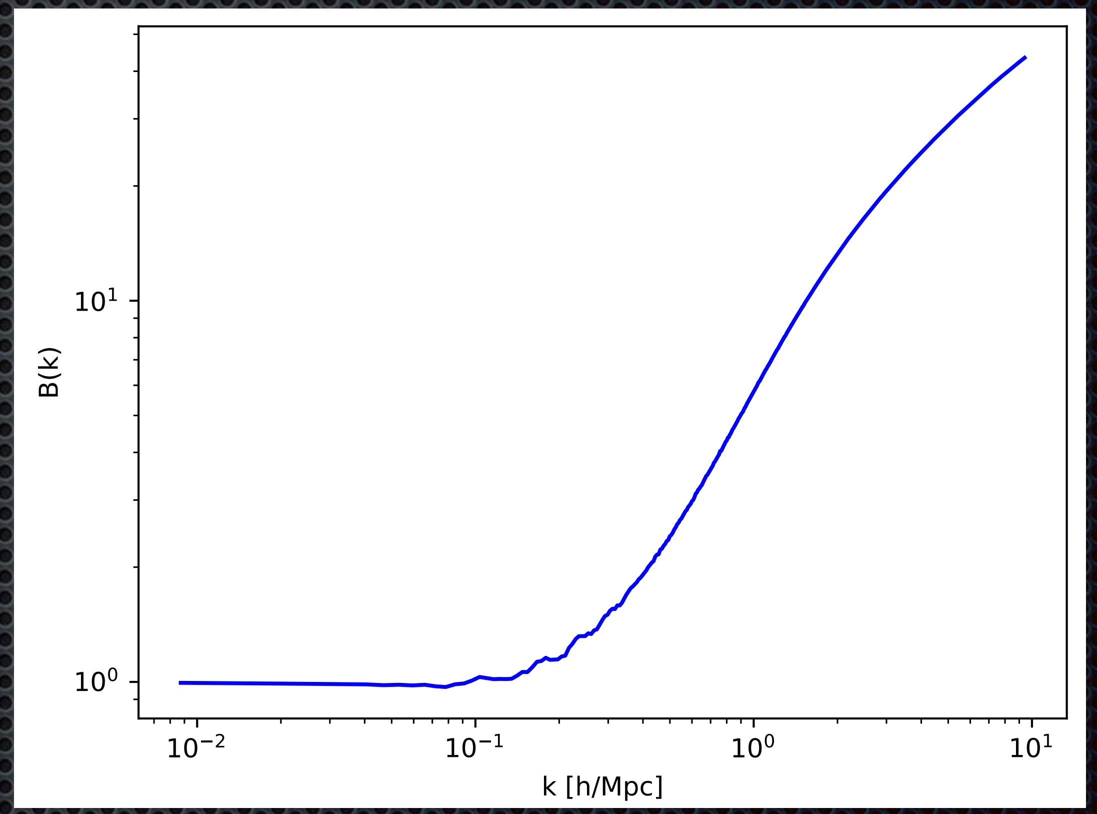
Cosmological parameters are the features

Statistics of the cosmic density field

$$\begin{pmatrix} \Omega_b \\ \Omega_m \\ \vdots \\ A_s \end{pmatrix} \xrightarrow{\text{Sim}}$$



post-processing → ⋯⋯ →



feature vector

statistical measure = target

What is the mapping [cosmo params → statistical measure]?

Two options:

Standard approach: Simulation (physics-driven)
Are very accurate but computationally demanding (HPC)

Alternative approach: Emulation (data-driven)
Still accurate but less demanding (HPC)

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We use polynomial chaos expansion

Polynomial chaos expansion (PCE)

Supervised regression technique based on orthonormal polynomials:

- PCE performs well even when trained on small data sets (!!!)
- PCE is interpretable (coefficients have a statistical interpretation)
- …

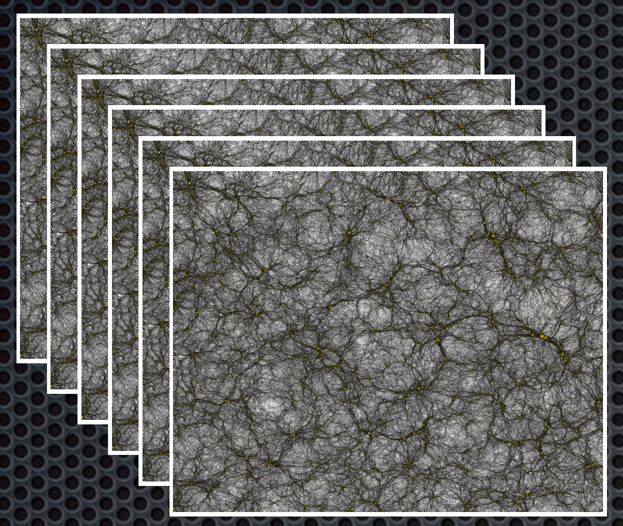
www.uqlab.com

The training pipeline

$$[\Omega_b^1, \Omega_m^1, \ldots, A_s^1]$$
$$[\Omega_b^2, \Omega_m^2, \ldots, A_s^2]$$
$$\vdots$$
$$[\Omega_b^n, \Omega_m^n, \ldots, A_s^n]$$

Sims

$\longrightarrow$



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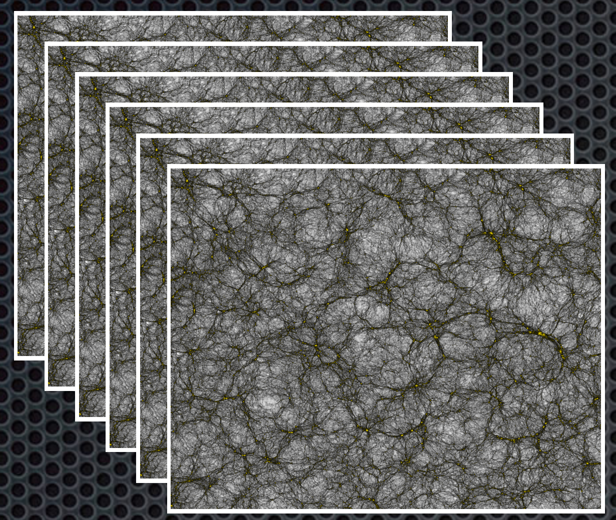
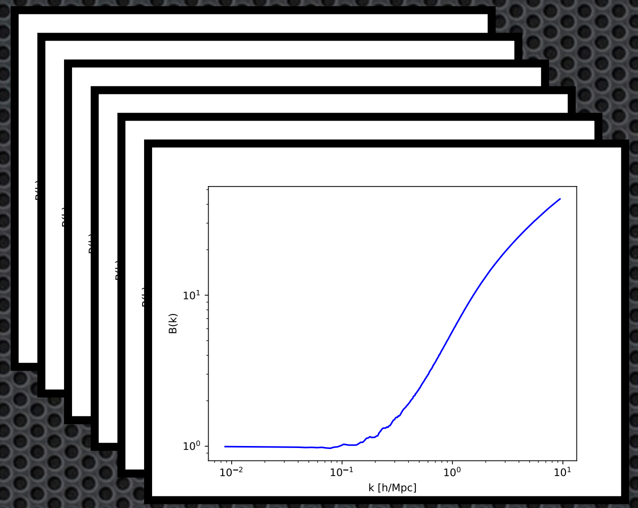


Stats $\longrightarrow$



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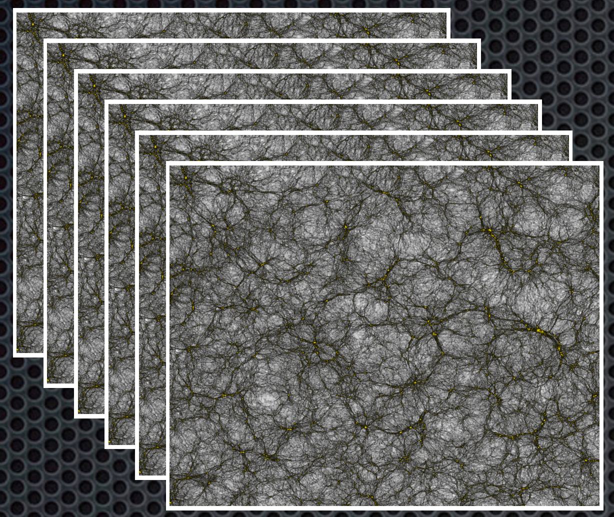

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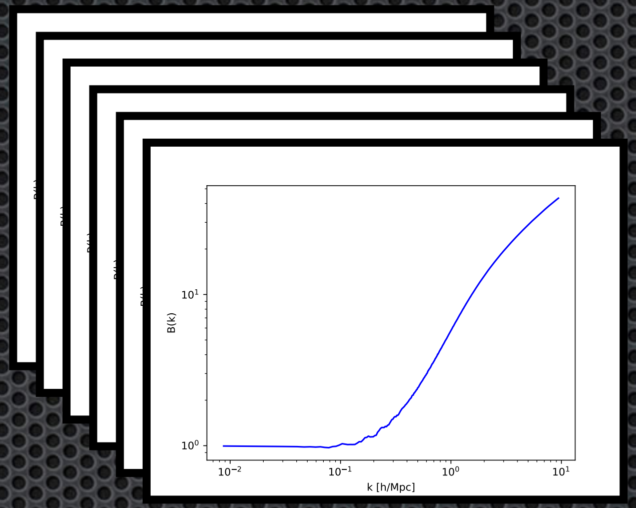
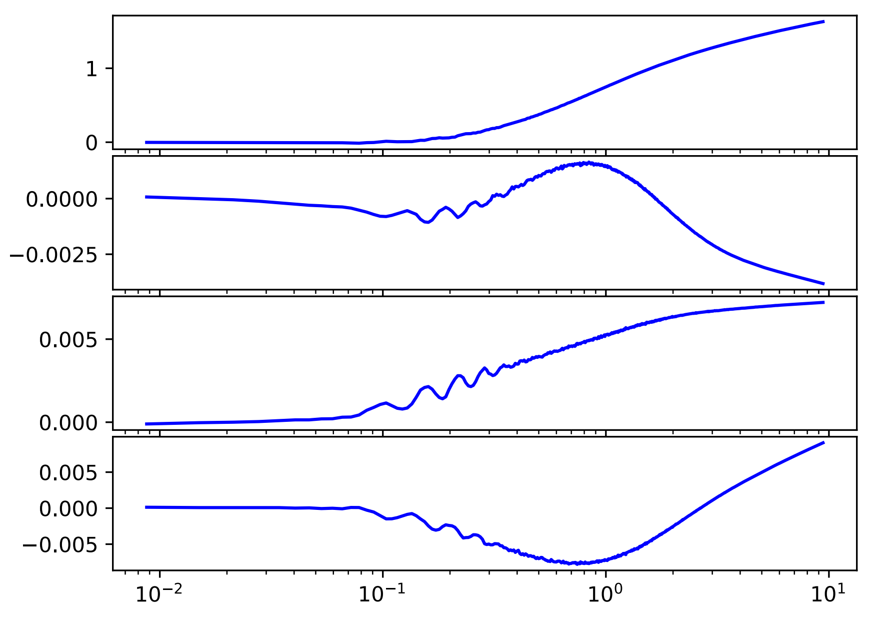


PCA $\longrightarrow$

$$\begin{pmatrix} 1 \\ w_1 \\ w_2 \\ w_3 \\ \vdots \end{pmatrix} \times$$



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$$[\Omega_b^1, \Omega_m^1, \dots, A_s^1]$$
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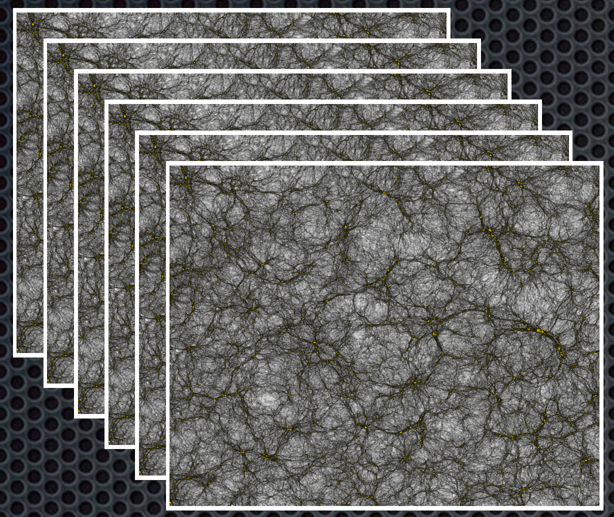

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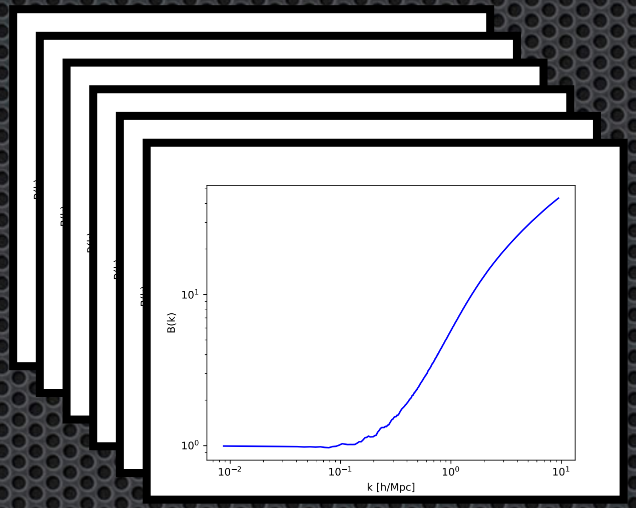

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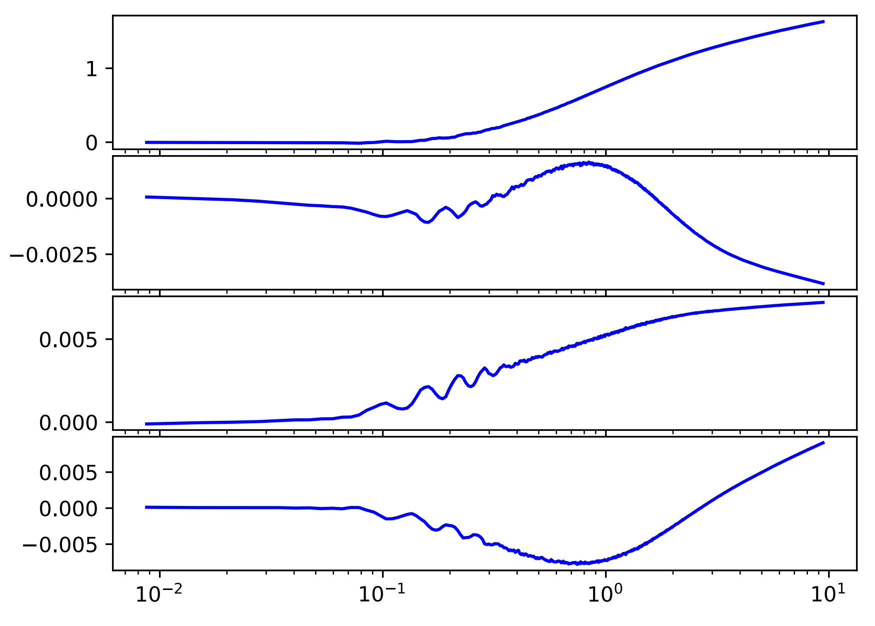


$$w_i(\Omega_b, \dots, A_s) = \sum_{\alpha_i \in \mathcal{A}^i} \eta_{\alpha_i}(\Omega_b, \dots, A_s)\Psi_{\alpha_i}$$

One PCE emulator per principal component (PC)!

EuclidEmulator specifications

EuclidEmulator1

(published)

- 6D feature space
- 11 PCs → 11 emulators
  (each with a 1D target)
- 100 training examples

EuclidEmulator2

(in prep.)

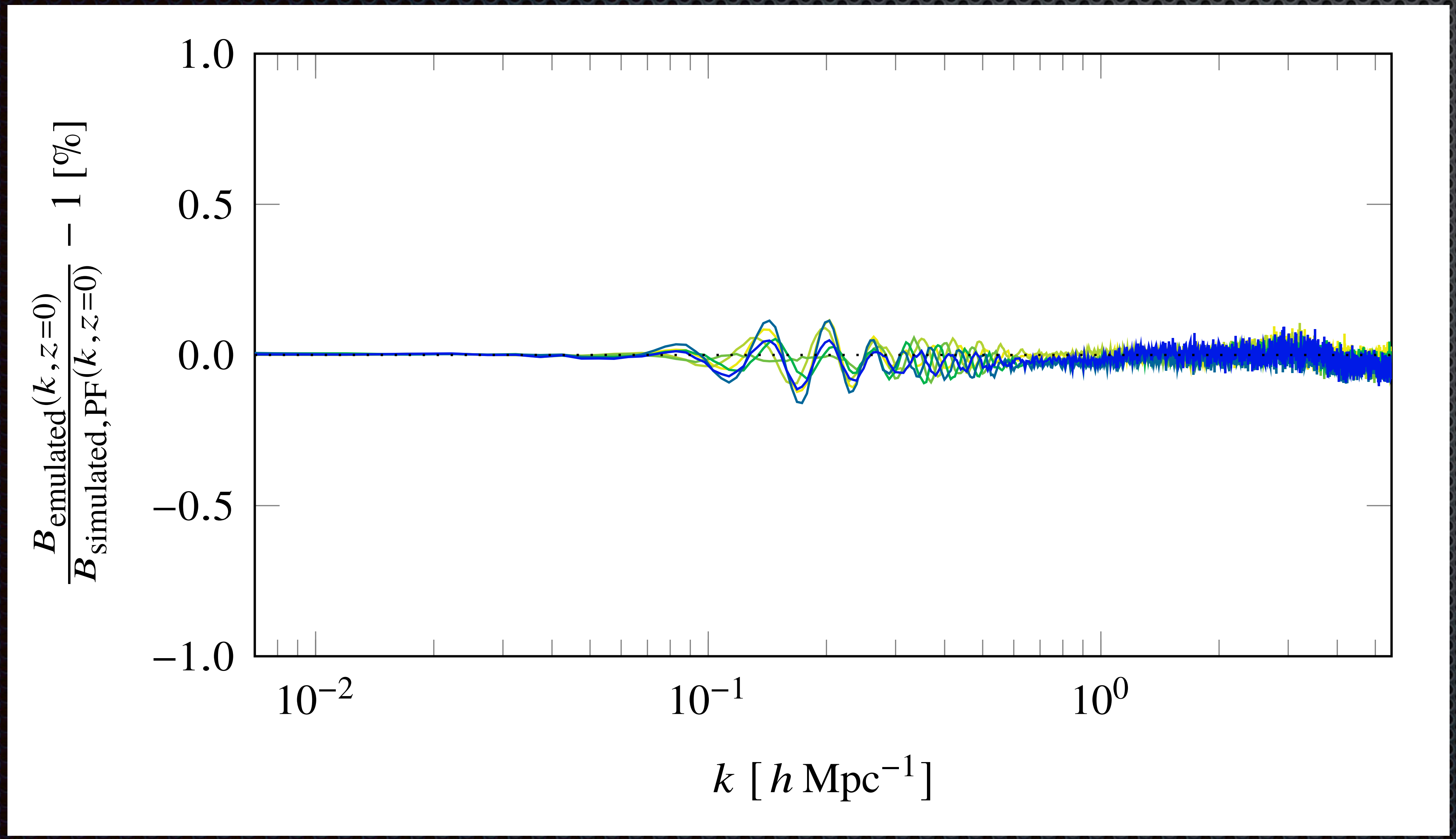
- 8D feature space
- 14 PCs → 14 emulators
  (each with a 1D target)
- 127 training examples

Performance (EuclidEmulator1)

Validation

36 validation examples

max validation error < 1%



Speed

$\mathcal{O}(10^{-5})$ CPU hours per evaluation

compared to

$\mathcal{O}(10^{4})$ CPU hours per simulation

Speed-up factor: 10^9

Exploiting the non-black-box nature

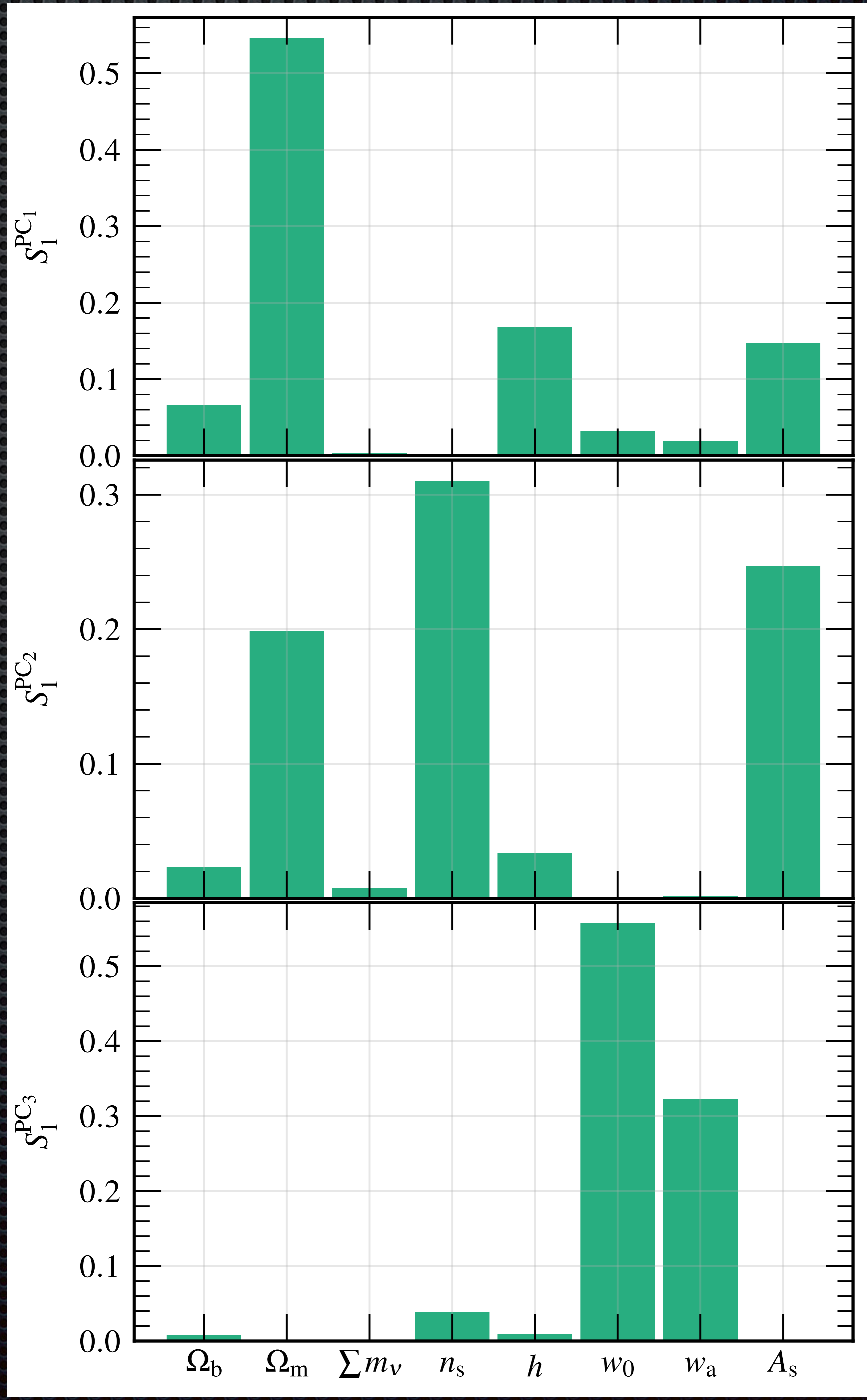
Sobol' analysis

Just by re-ordering expansion terms

we get a sensitivity analysis

(for free - no sampling required!)

→ **Very valuable e.g. for feature selection!**



Find me online…

https://github.com/miknab/EuclidEmulator

https://arxiv.org/abs/1809.04695

https://www.linkedin.com/in/mischa-knabenhans/

Thank you…

Further reading

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S. Marelli and B. Sudret, ICVRAM 2014, University of Liverpool (UK), July 13–16, 2014, pp. 2554–2563

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B. Efron et al. (2004), The Annals of Statistics, 32 (2), 407-499