Quantifying of geometric measure of entanglement of quantum Generative Adversarial Network states on a quantum computer

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Outline

- Relation of the geometric measure of entanglement with mean spin.
- Geometric measure of entanglement of quantum Generative Adversarial Network states.
- Detection of the entanglement on IBM's quantum computer.
- Conclusions.

Geometric measure of entanglement and its relation with mean spin

Geometric measure of entanglement is defined as minimal squared Fubiny-Study distance between an entangled state $\mid \psi\rangle$ and a set of non-entangled states $\mid \psi_s\rangle$

$$E(\mid \psi\rangle) = \min_{|\psi_s\rangle}(1 - |\langle\psi|\psi_s\rangle|^2). \quad (1)$$

[A. Shimony, Ann. N.Y. Acad. Sci. 755, 675 (1995)]. The geometric measure of entanglement of a spin with quantum system in a state

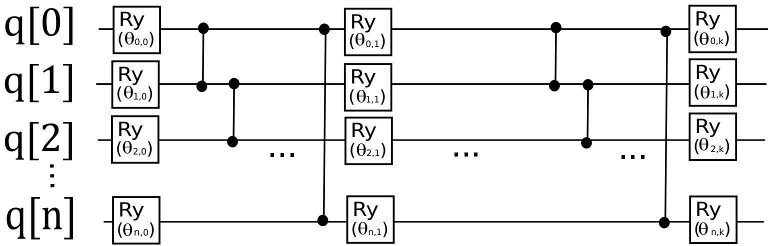
$$\mid \psi\rangle = a\,|\uparrow\rangle \mid \Phi_1\rangle + b\,|\downarrow\rangle \mid \Phi_2\rangle, \quad \langle\Phi_i \mid \Phi_i\rangle = 1, \quad (2)$$

(a, b are constants, $\mid \Phi_1\rangle$, $\mid \Phi_2\rangle$ are states of a quantum system) is related with the mean value of spin as

$$E(\mid \psi\rangle) = \frac{1}{2}(1 - |\langle\boldsymbol{\sigma}\rangle|), \quad \langle|\boldsymbol{\sigma}|\rangle = \sqrt{\langle\boldsymbol{\sigma}\rangle}. \quad (3)$$

Result (3) was obtained in [A. M. Frydryszak, M. I. Samar, V. M. Tkachuk, Eur. Phys. J. D 71, 233 (2017)].

Geometric measure of entanglement of quantum Generative Adversarial Network states



The variational n-qubit form corresponding to quantum generator. The layer is formed by the rotational gates $RY(\theta_{i,j})$ and two-qubit CZ gates, k is the depth [Ch. Zoufal, A. Lucchi, S. Woerner, npj Quantum Information 5, 103 (2019)].

Quantum state generated by the rotational gates and the entangling blocks

$$\mid G\rangle = \prod_{(a,b)\in E} CZ_{ab} \prod_{i} RY_i(\theta_{i,0}) \mid 0\rangle^{\otimes n}, \tag{4}$$

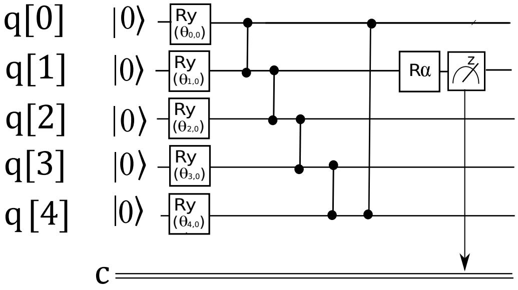
can be associated with the undirected graph $G(E,V)$ with the structure of ring. The geometric measure of entanglement of a qubit with other qubits in graph state (4) is related with the graph properties. The entanglement of qubit $q[l]$ with other qubits is related with the degree of vertex n_l representing it. In the case of $\theta_{i,0} = \theta$, it reads

$$E_l = \frac{1}{2} - \frac{1}{2}\sqrt{\sin^2\theta\,(\cos^2\theta)^{n_l} + \cos^2\theta}. \tag{5}$$

[Kh. P. Gnatenko, N. A. Susulovska, EPL 136, 40003 (2021), Kh. P. Gnatenko, V. M. Tkachuk, Phys. Lett. A. 396, 127248 (2021).]

Detection of the entanglement on IBM's quantum computer

Quantum protocol for detection of the geometric measure of entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 1$



In the protocol we take into account that

$$\langle\sigma^x\rangle = \langle\psi|\sigma^x|\psi\rangle = \langle\tilde{\psi}^y|\sigma^z|\tilde{\psi}^y\rangle = |\langle\tilde{\psi}^y|0\rangle|^2 - |\langle\tilde{\psi}^y|1\rangle|^2, \tag{6}$$

$$|\tilde{\psi}^y\rangle = \exp(\mathrm{i}\pi\sigma^y/4)|\psi\rangle, \tag{7}$$

$$\langle\sigma^y\rangle = \langle\psi|\sigma^y|\psi\rangle = \langle\tilde{\psi}^x|\sigma^z|\tilde{\psi}^x\rangle = |\langle\tilde{\psi}^x|0\rangle|^2 - |\langle\tilde{\psi}^x|1\rangle|^2, \tag{8}$$

$$|\tilde{\psi}^x\rangle = \exp(-\mathrm{i}\pi\sigma^x/4)|\psi\rangle. \tag{9}$$

Therefore to quantify the mean value $\langle\sigma^x\rangle$ the

$$R_\alpha = RY(-\pi/2)$$

gate has to be applied to the state of a qubit before measurement. To detect the mean value $\langle\sigma^y\rangle$ the

$$R_\alpha = RX(\pi/2)$$

gate has to be used.

Results of calculations of the entanglement, $k = 1$

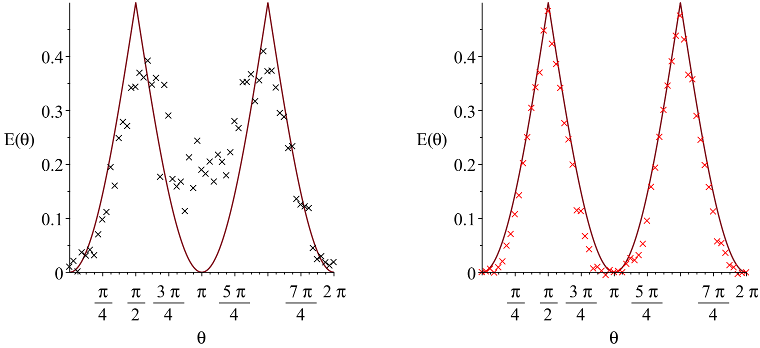
Results of calculations of the entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 1$

$$E_1(\theta_{0,0}, \theta_{1,0}, \theta_{2,0}) = \frac{1}{2}(1 - |\langle \boldsymbol{\sigma}_1 \rangle|) =$$
$$= \frac{1}{2}\left(1 - \sqrt{\cos^2\theta_{0,0}\cos^2\theta_{2,0}\sin^2\theta_{1,0} + \cos^2\theta_{1,0}}\right). \qquad (10)$$

In the case when $\theta_{i,0} = \theta$ the entanglement reads

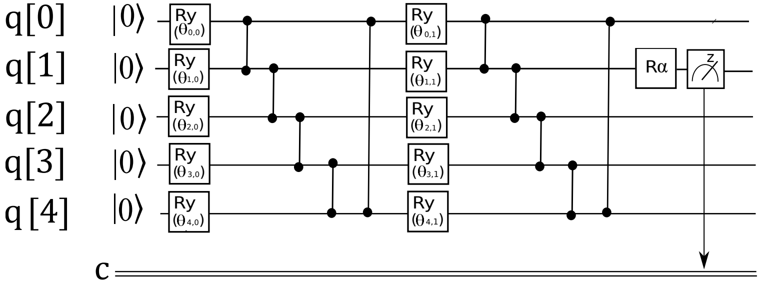
$$E_1(\theta, \theta, \theta) = \frac{1}{2}(1 - \sqrt{\cos^4\theta\sin^2\theta + \cos^2\theta}). \qquad (11)$$

Results of quantum calculations of the geometric measure of entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 1$ for different values of $\theta_{i,0} = \theta$ on ibmq-manila (marked by black crosses) (a), ibmq-qasm-simulator (red crosses) (b), and analytical results (line)



(a) (b)

Quantum protocol for detection of the geometric measure of entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 2$



Results of calculations of the entanglement, $k = 2$

Results of calculations of the entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 2$ in the case of $\theta_{i,0} = \pi/2$

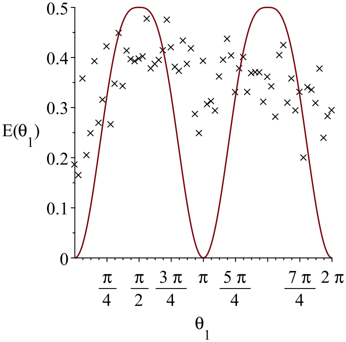
$$E(\theta_{0,1}, \theta_{1,1}, \theta_{2,1}) = \frac{1}{2}(1 - |\langle \boldsymbol{\sigma}_1 \rangle|) =$$

$$= \frac{1}{2}(1 - \frac{1}{4}|\cos(\theta_{0,1} + \theta_{1,1} - \theta_{2,1}) + \cos(\theta_{0,1} - \theta_{1,1} + \theta_{2,1}) +$$
$$+ \cos(-\theta_{0,1} + \theta_{1,1} + \theta_{2,1}) + \cos(\theta_{0,1} + \theta_{1,1} + \theta_{2,1})|). \quad (12)$$

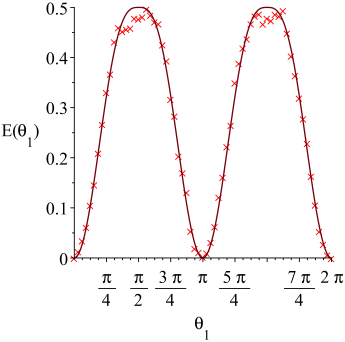
For $\theta_{i,1} = \theta_1$ the expression is reduced as

$$E(\theta_1, \theta_1, \theta_1) = \frac{1}{2}(1 - |\cos^3 \theta_1|). \quad (13)$$

Results of calculations of entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 2$ for different values of $\theta_{i,1} = \theta_1$ and $\theta_{i,0} = \pi/2$ obtained on ibmq-manila (black crosses) (c), ibmq-qasm-simulator (red crosses) (d), and analytical results (line)



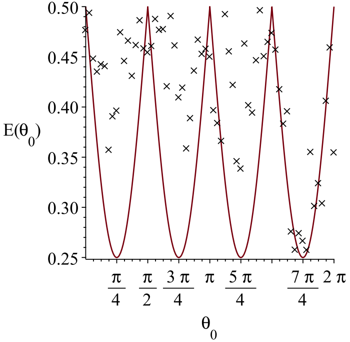
(c)

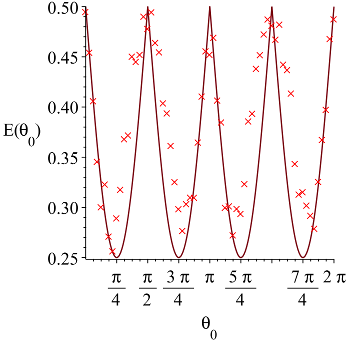


(d)

Results of calculations of entanglement of qubit $q[1]$ with other qubits in a state generated by quantum generator with depth $k = 2$ for different values of $\theta_{i,0} = \theta_0$ and $\theta_{i,1} = \pi/2$ obtained on ibmq-manila (black crosses) (a), ibmq-qasm-simulator (red crosses) (b), and analytical results (line)

$$E(\theta_0, \theta_0, \theta_0) = \frac{1}{2} - \frac{1}{4}|\sin(2\theta_0)|.$$





Conclusions

- Geometric measure of entanglement of a qubit with other qubits in a states of quantum Generative Adversarial Network has been examined.
- The entanglement in a quantum graph state prepared by the quantum generator with depth k=1 is related to the degree of the vertex corresponding to the qubit in the graph.
- The entanglement of qubit $q[i]$ with other qubits in the states prepared by the quantum generator is determined by the parameters $\theta_{i,a}$, $\theta_{j,a}$ of RY gates in the variational form that act on the qubits entangled with $q[i]$ by CZ_{ij} gates (index a corresponds to the depth).
- Entanglement of states generated by quantum generator has been quantified on IBM's quantum computer ibmq-manila.

Thank you for your attention!

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