Quantum State Reconstruction with Artificial and Spiking Neural Networks

Stefanie Czischek

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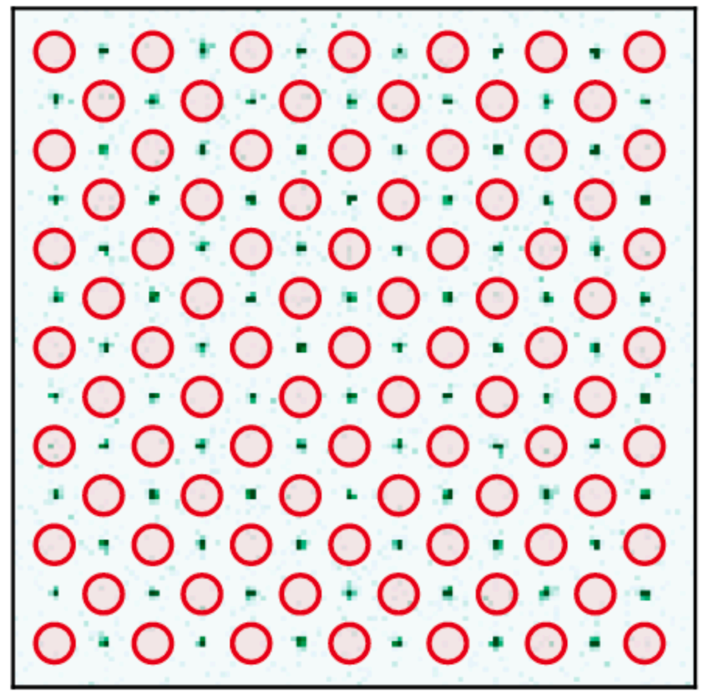
UNIVERSITY OF WATERLOO

PIQUIL

Quantum Computation and Simulation

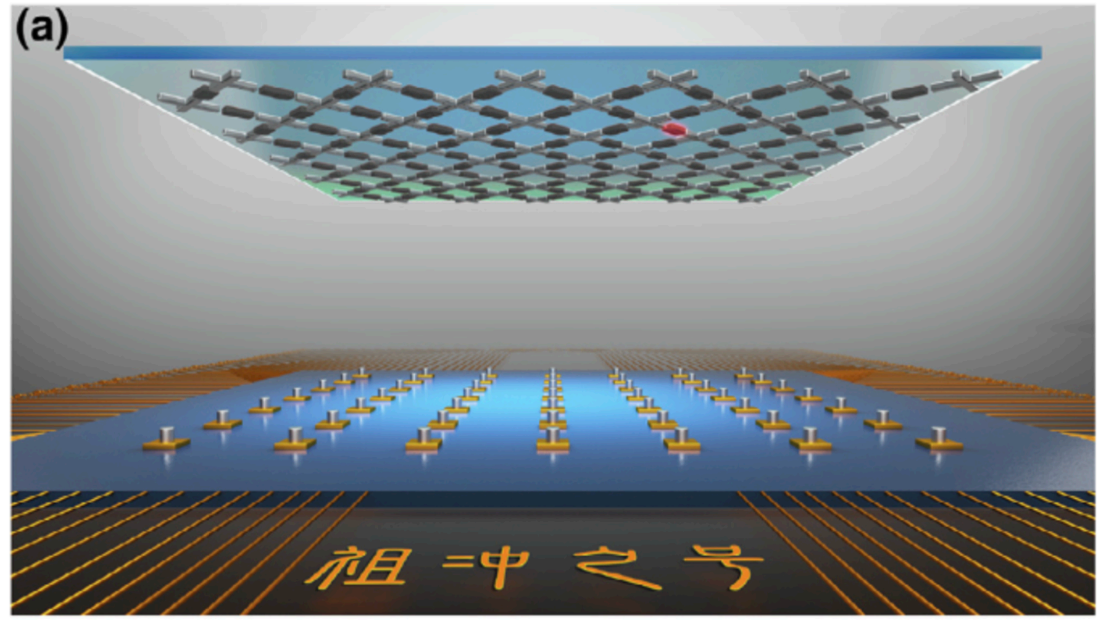
- Growing number of qubits
- Presice control
- Individual addressing
- High-quality preparation of target quantum states

Rydberg atoms



[S. Ebadi et al., Nature **595** (2021)]

Superconductors



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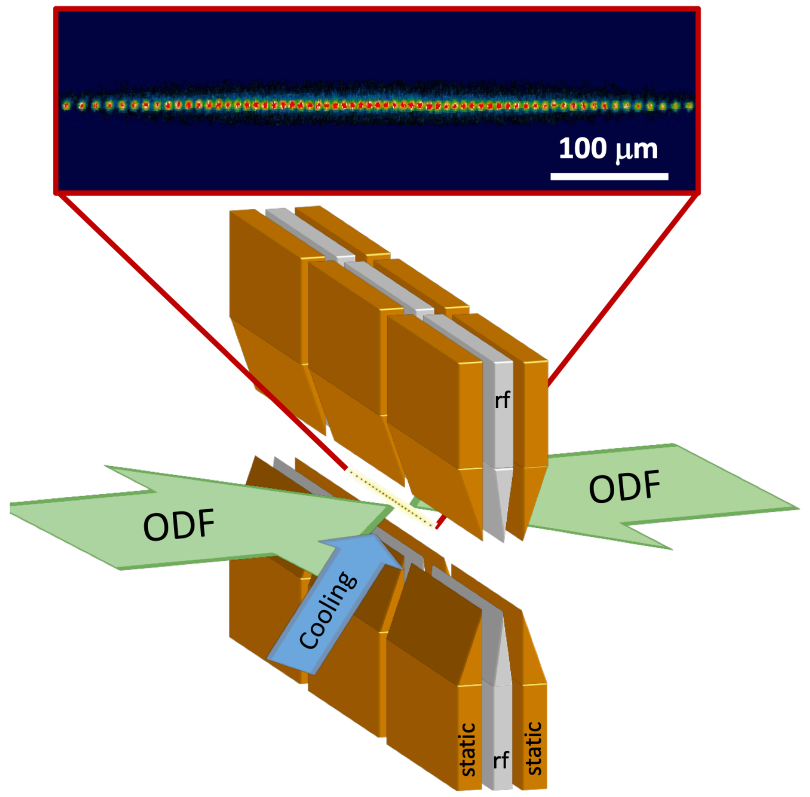
Trapped ions



[C. Monroe et al., Rev. Mod. Phys **93** (2021)]

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What can we do with those prepared states?

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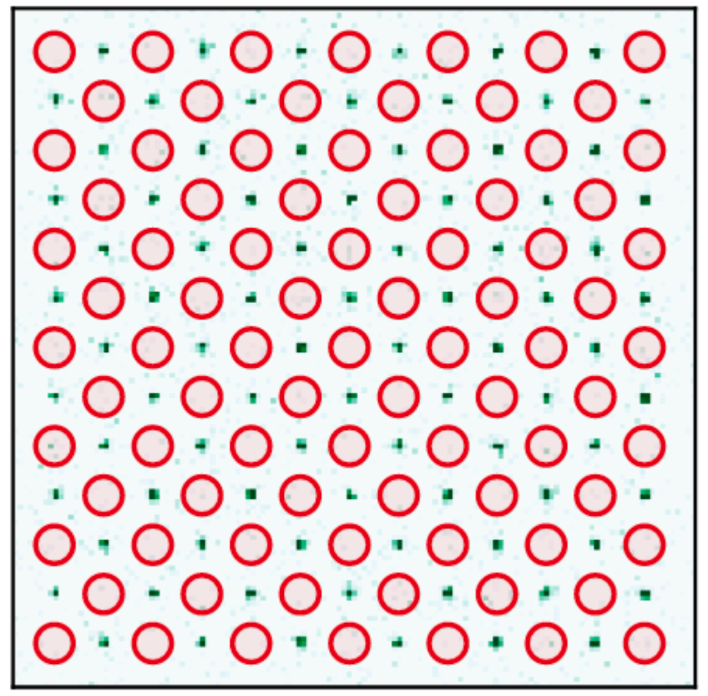

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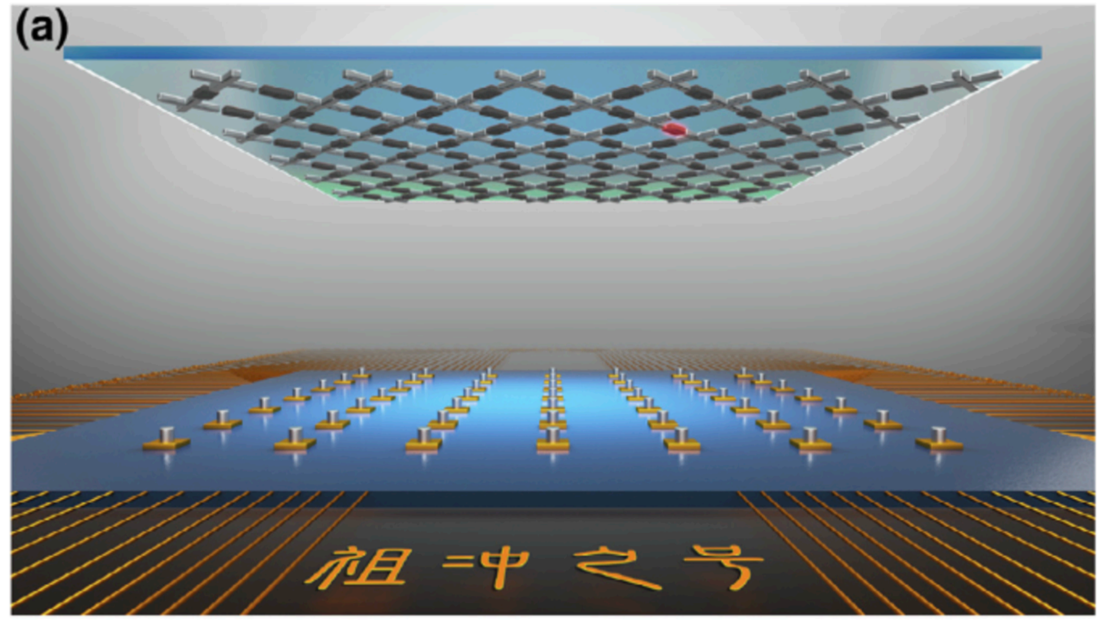

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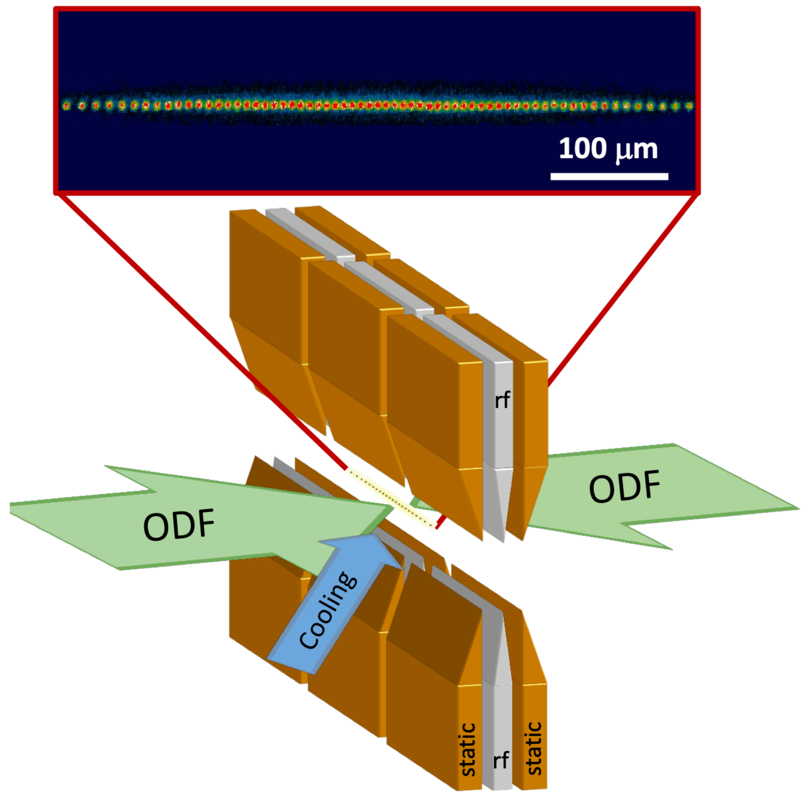

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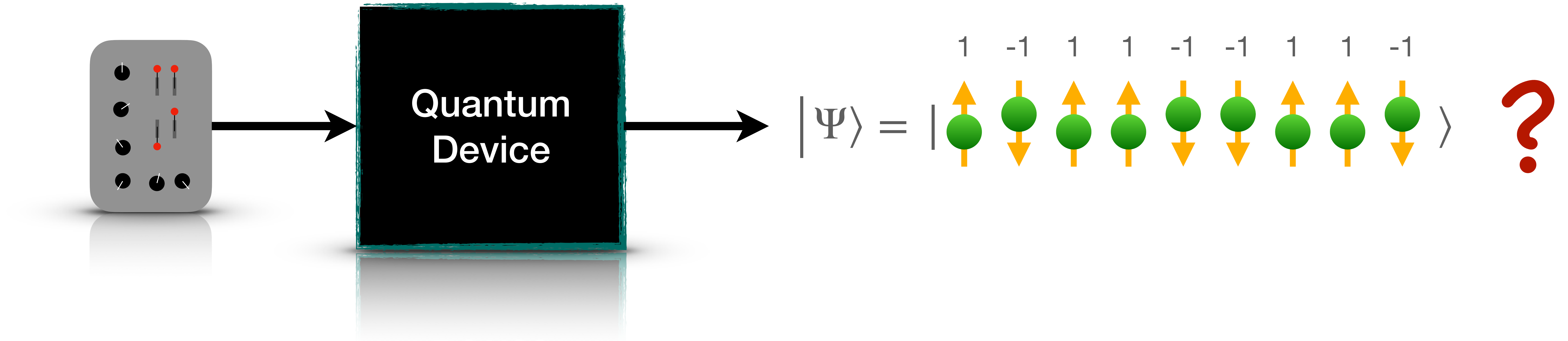
Quantum State Reconstruction



- Perform limited amount of measurements
- Tomographically reconstruct quantum state
  - Generate more measurement data

[G. Torlai et al., PRR **2** (2020)]

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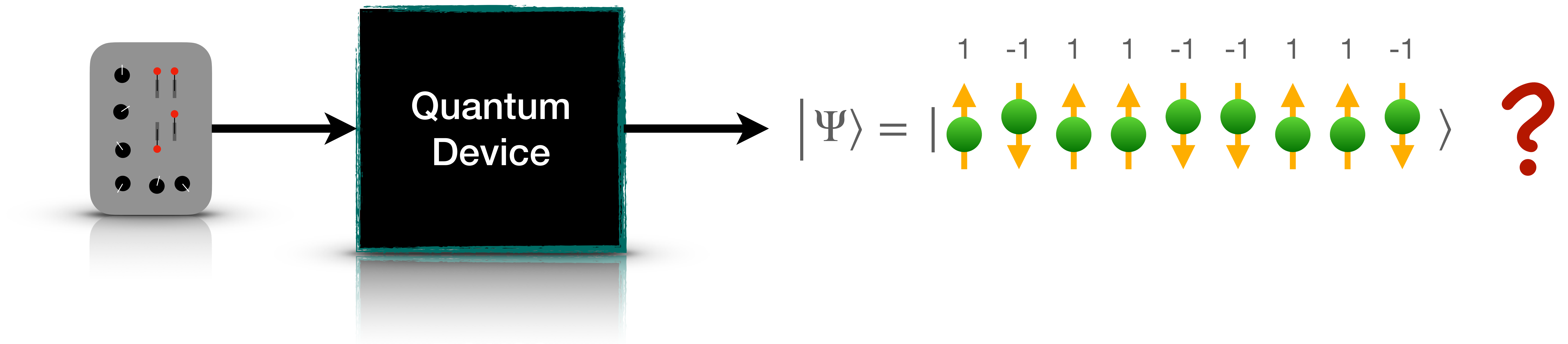

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Generally:

$$|\Psi\rangle = \sum_{\{\varphi\}} \Psi(\varphi)|\varphi\rangle \in \mathbb{C}^{2^N}$$ (wavefunction)

$$\Psi(\varphi) \in \mathbb{C}$$ (amplitude)

$$\varphi \in \{(-1,-1,\ldots,-1),(-1,-1,\ldots,-1,1),\ldots,(1,1,\ldots,1)\}$$ (basis states)

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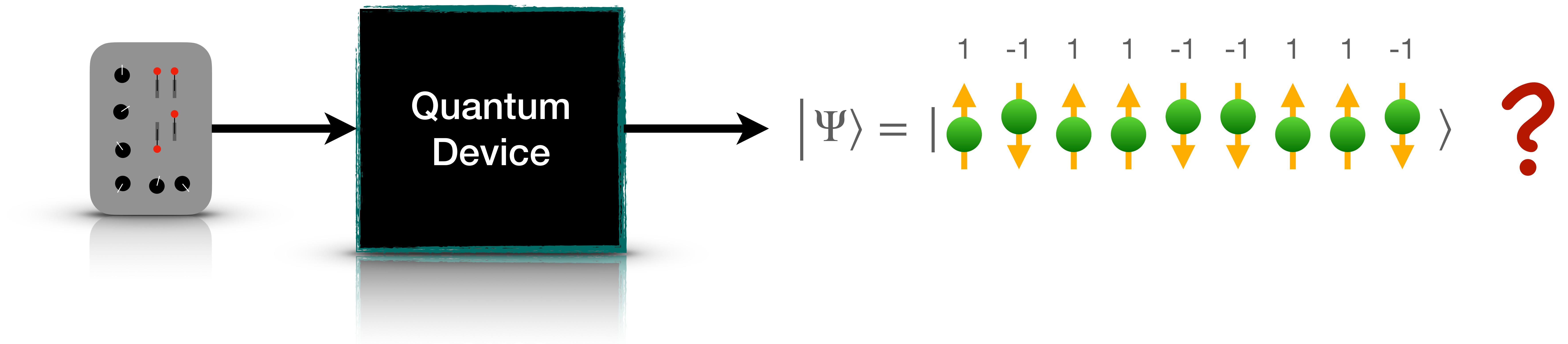
Quantum State Reconstruction



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$$\langle \mathcal{O} \rangle = \left\langle \Psi \middle| \mathcal{O} \middle| \Psi \right\rangle \qquad \langle \mathcal{O} \rangle = \sum_{\{\varphi,\tilde{\varphi}\}} \langle \varphi | \, \mathcal{O} \, | \tilde{\varphi} \rangle \, \Psi(\tilde{\varphi}) \Psi(\varphi)^*$$

$$\left\langle \mathcal{O}_{\text{diag}} \right\rangle = \sum_{\{\varphi\}} \mathcal{O}_{\text{diag}}(\varphi) \, \Psi(\varphi) \Psi(\varphi)^*$$

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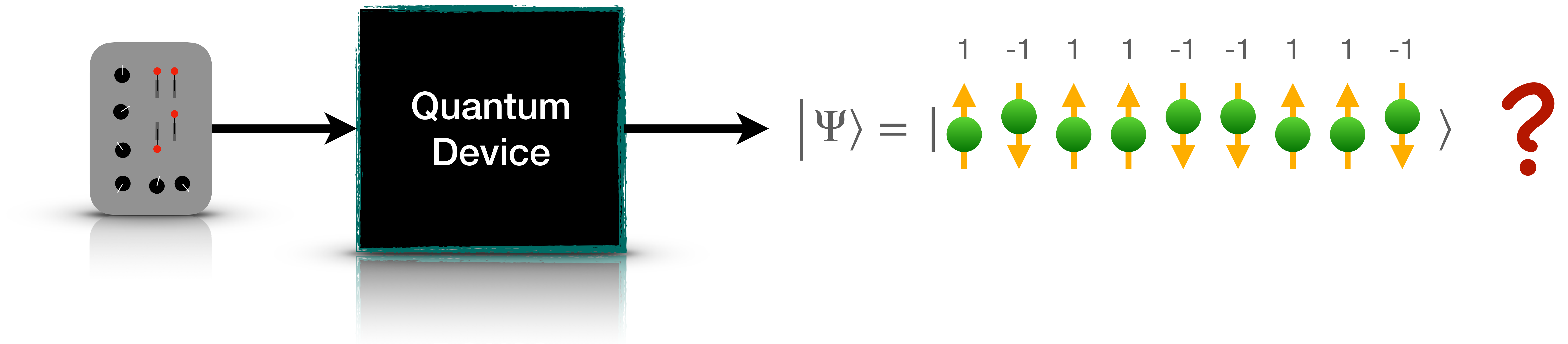
Quantum State Reconstruction



- Perform limited amount of measurements
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  - Generate more measurement data
    [G. Torlai et al., PRR **2** (2020)]
- Find efficient expression for $\Psi(\boldsymbol{\varphi})$

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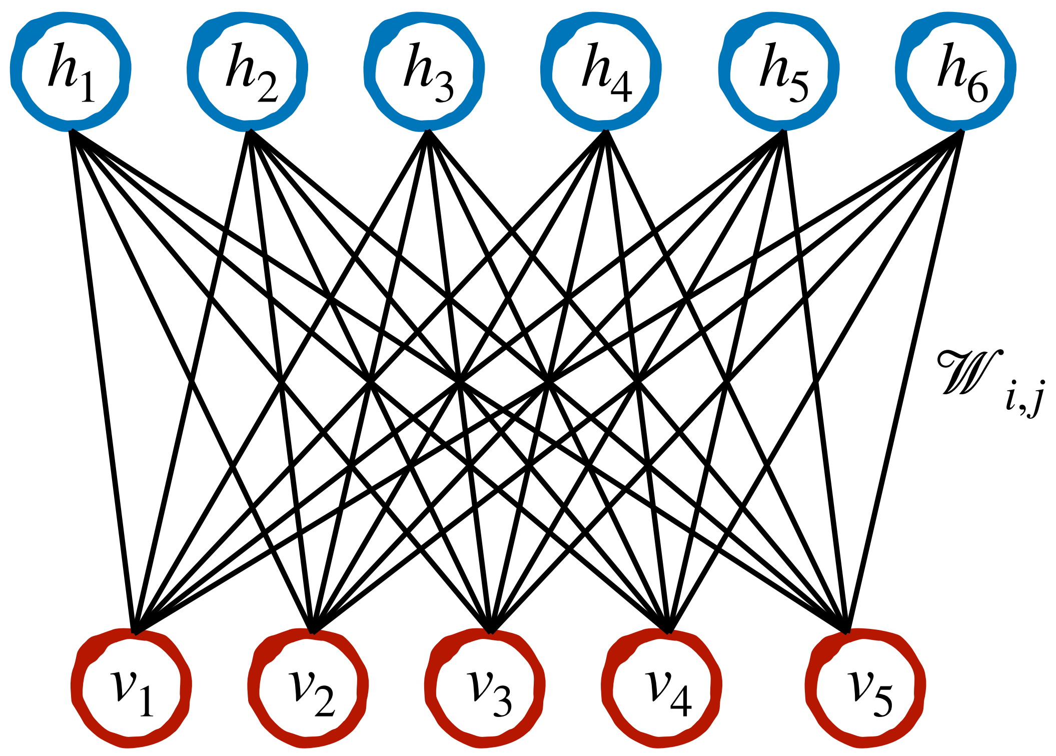
Quantum State Reconstruction with ANNs

Restricted Boltzmann machine 🦑



$$E\left[\boldsymbol{v},\boldsymbol{h};\mathscr{W}\right] = -\sum_{i,j} v_i \mathscr{W}_{i,j} h_j$$

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M hidden neurons $h_j \in \{-1,1\}$

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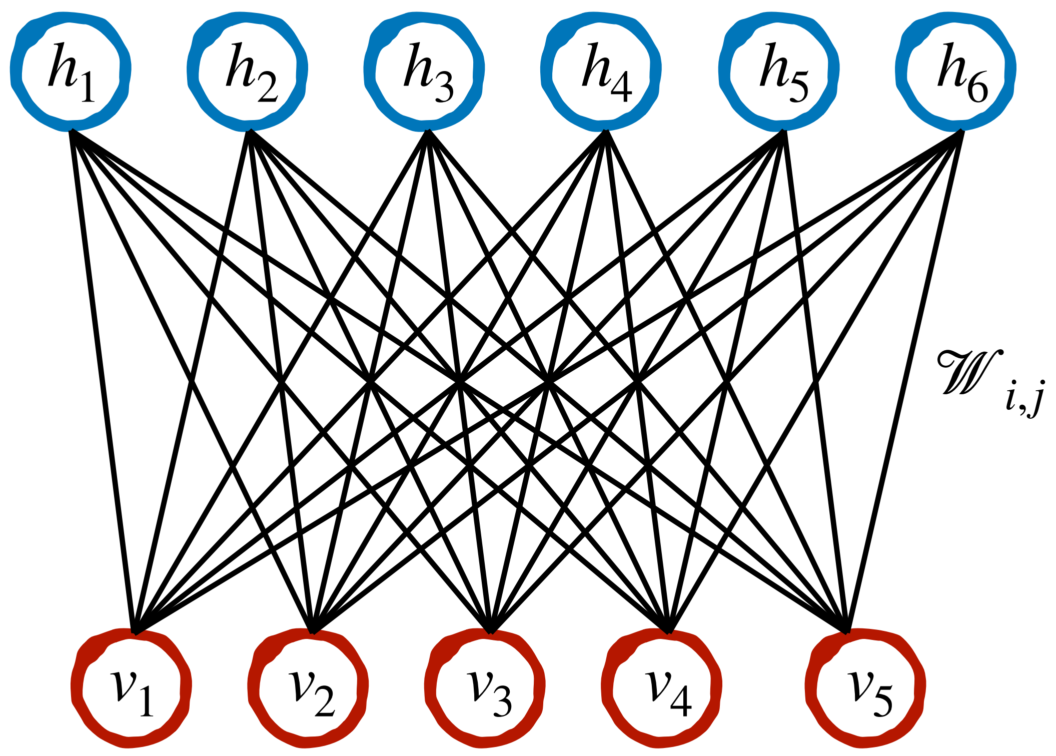

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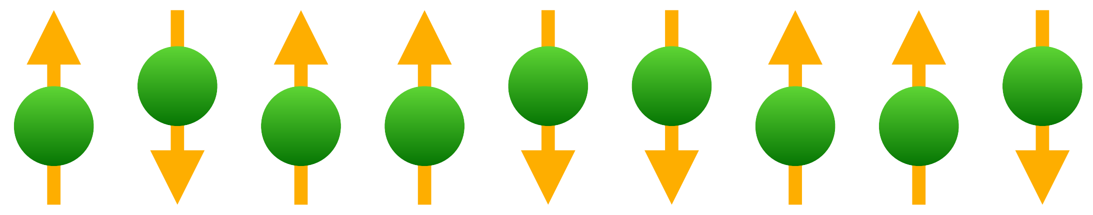
Quantum state

$\left|\Psi\right\rangle = \big|$  $\big\rangle$

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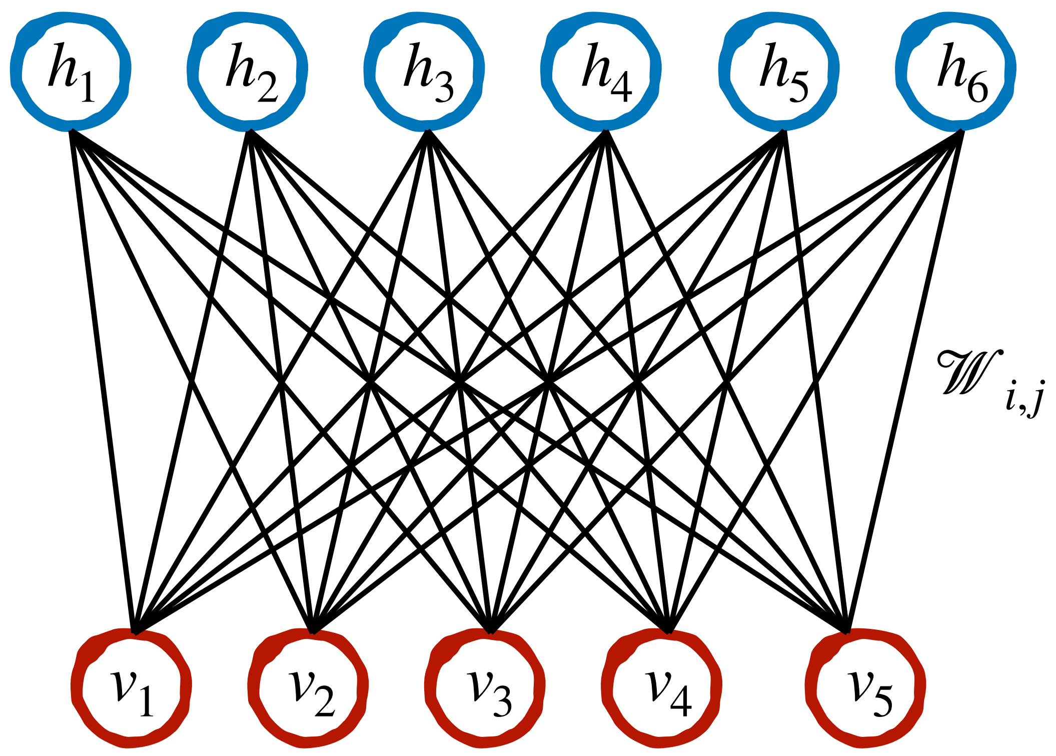

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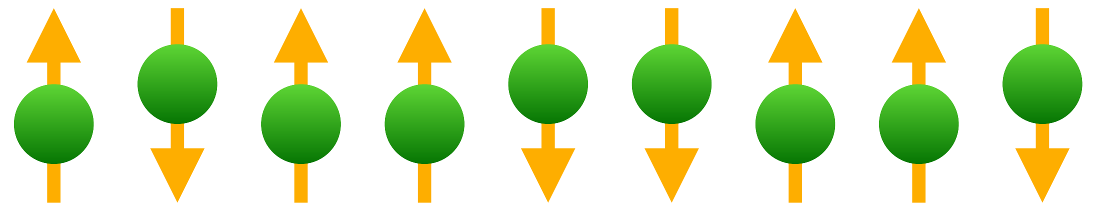
$|\Psi\rangle = |$  $\rangle$

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Train the restricted Boltzmann machine such that

$$P_{\mathcal{W}}\left[\boldsymbol{v}\right] \approx \Psi(\boldsymbol{v})\Psi(\boldsymbol{v})^*$$

Full information only if

$$\text{Im}\left[\Psi(\boldsymbol{v})\right] = 0 \; \forall \boldsymbol{v}$$

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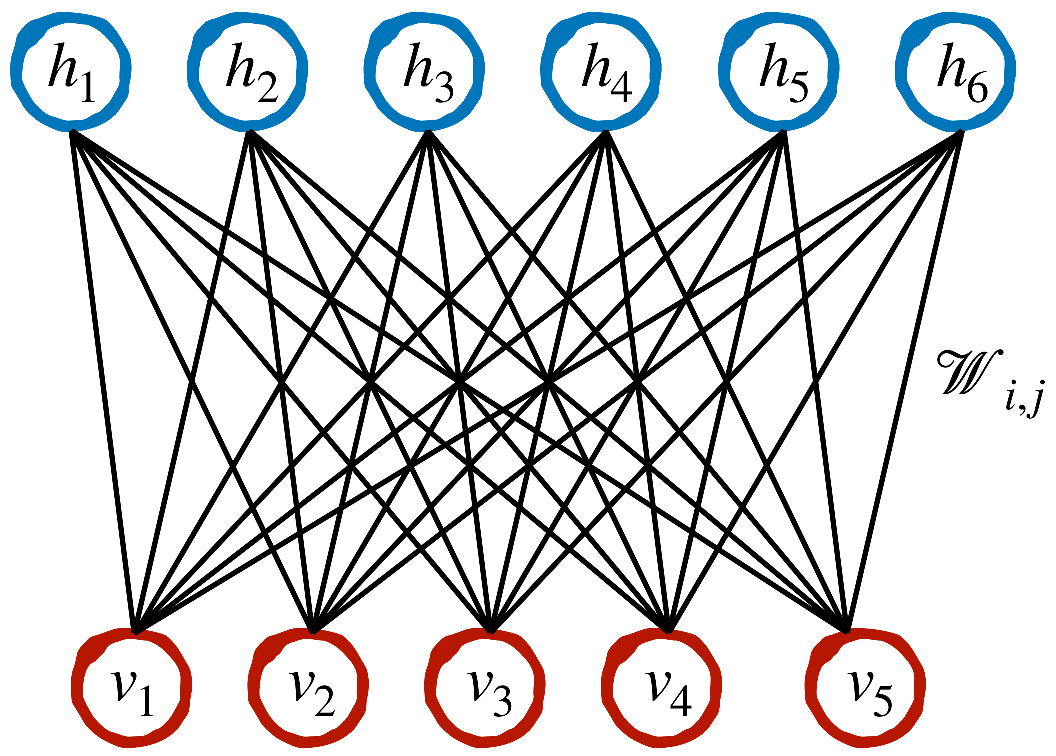

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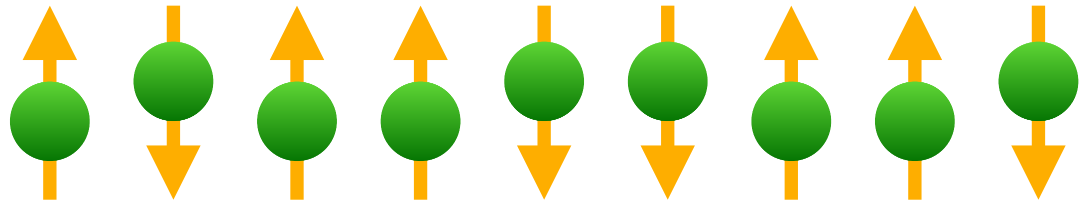
$|\Psi\rangle = |$  $\rangle$

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True for stoquastic Hamiltonians (e.g. Rydberg atoms)

[SC et al., arXiv:2203.04988 (2022)]
[J. Carrasquilla and G. Torlai, PRX Quantum **2** (2021)]

$$\hat{H} = \begin{pmatrix} d & -|o| & -|o| & -|o| \\ -|o| & d & -|o| & -|o| \\ -|o| & -|o| & d & -|o| \\ -|o| & -|o| & -|o| & d \end{pmatrix}$$

Otherwise modify the network architecture

[G. Torlai, R.G. Melko, PRL **120** (2018)] [G. Carleo, M. Troyer, Science **355** (2017)]

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Efficient approximation on spiking neuromorphic hardware!

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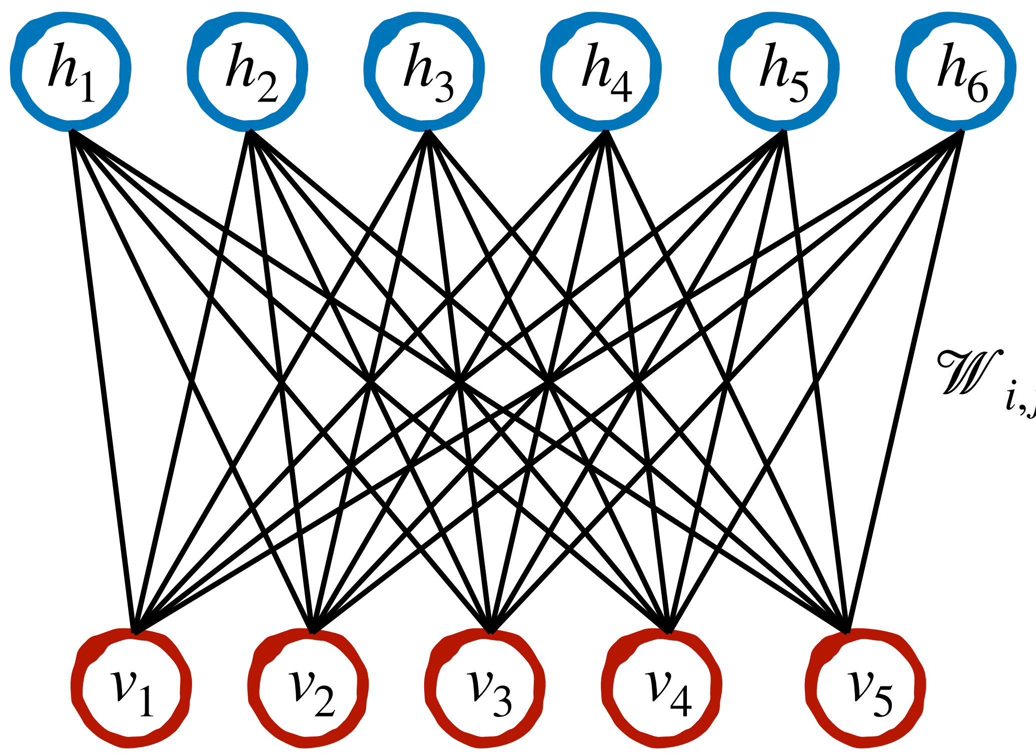

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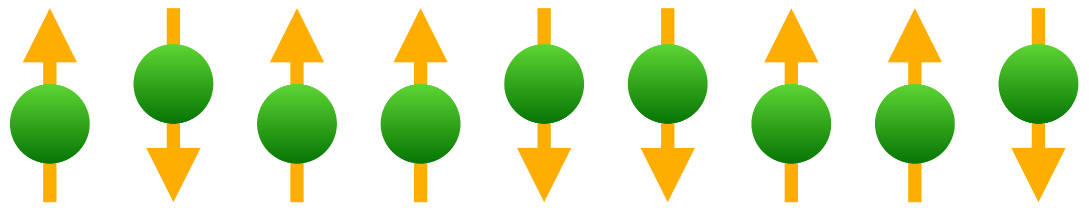
$|\Psi\rangle = |$  $\rangle$

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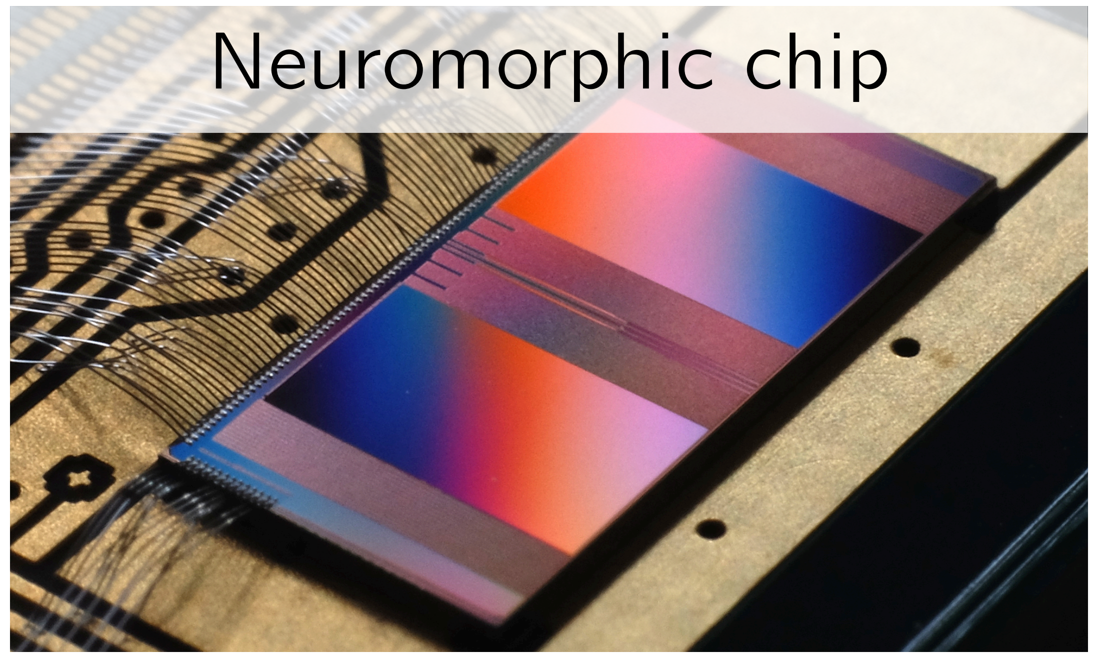
Spiking Neuromorphic Hardware



- Biological neural networks: Fast and energy-efficient
- Dynamical behaviour

- Capture biological properties
- Analog electronic circuits for efficient emulation

- BrainScaleS (Heidelberg University)
- SpiNNaker (University of Manchester)
- Loihi (Intel)

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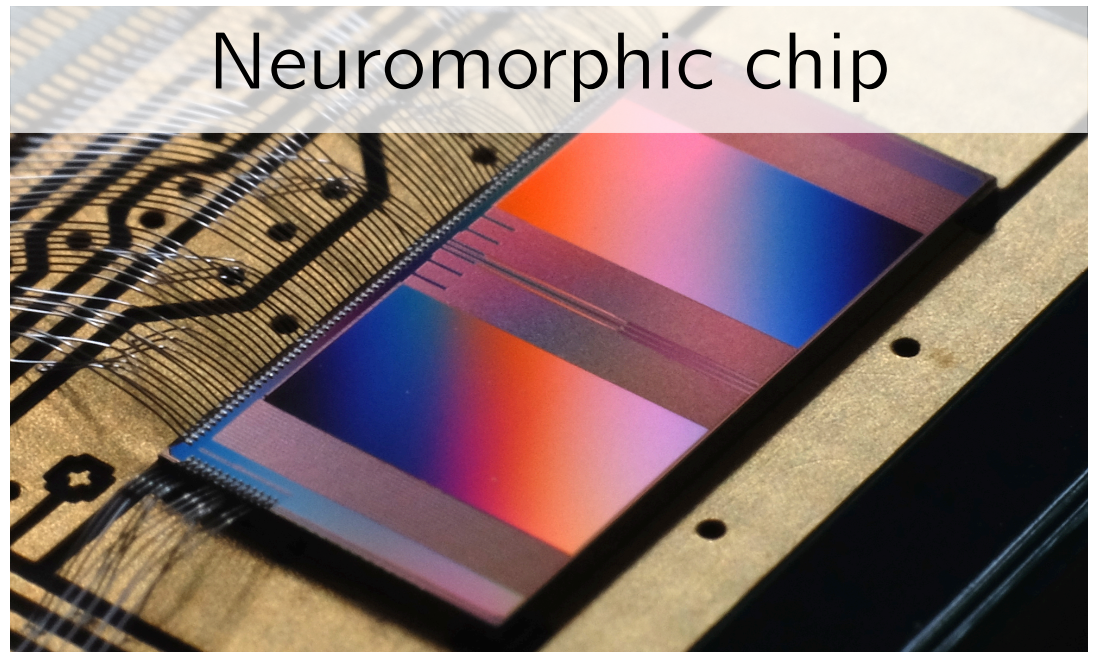

BrainScaleS

- Biological neural networks: Fast and energy-efficient
- Dynamical behaviour

Quantum state reconstruction with neuromorphic hardware:

- Generate a huge amount of data
- Integrate small chips in experimental setup

- Capture biological properties
- Analog electronic circuits for efficient emulation

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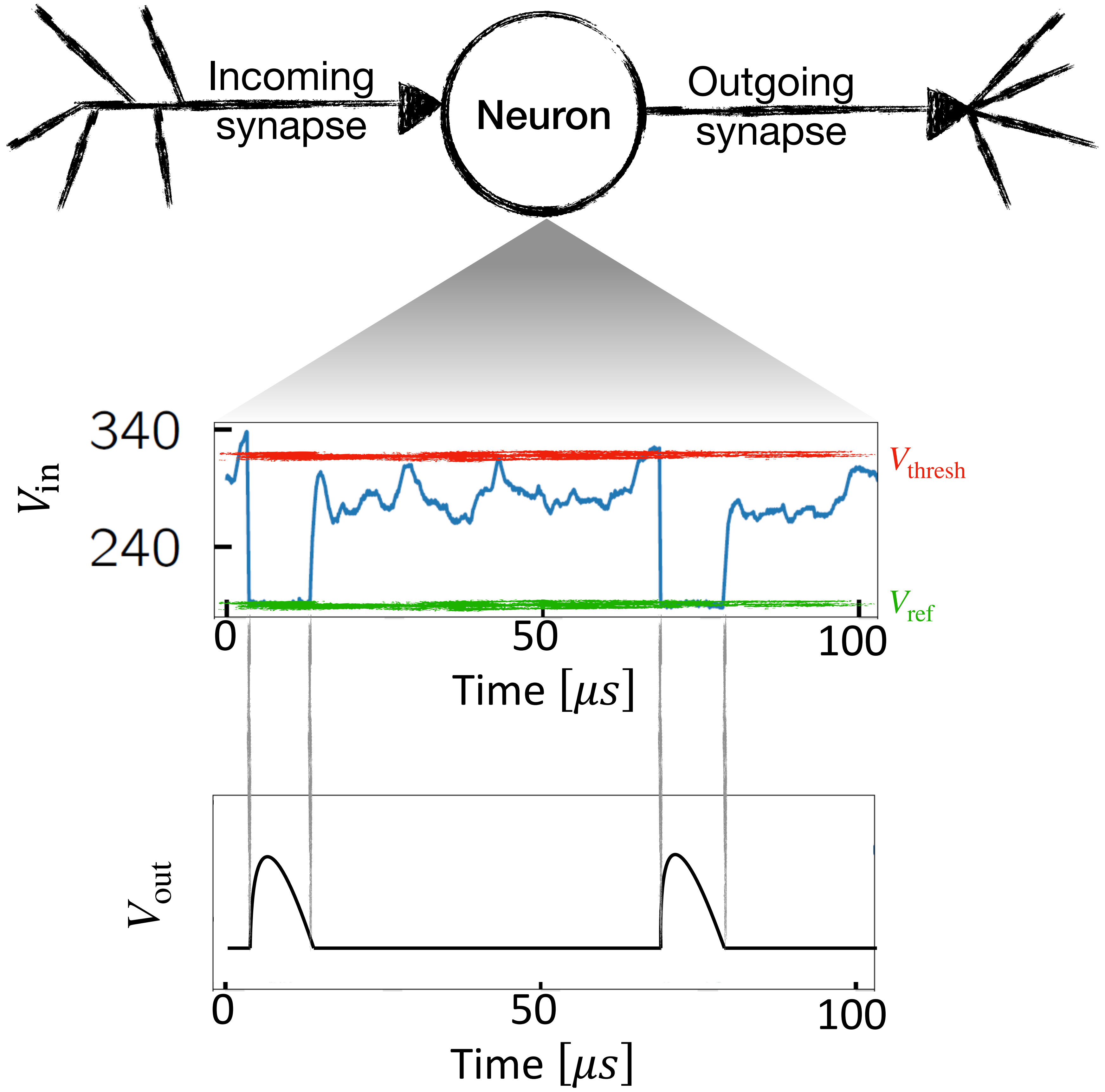
Spiking Neurons



$$V_{\text{in}}(i) \propto \sum_j V_{\text{out}}(j)\, W_{i,j}$$

- Leaky Integrate-and-Fire (LIF) neurons mimic biological neurons
- Potential V_{in} evolves in time
- V_{thresh}: neuron sends out spike V_{out} and becomes refractory
- Neuron connections are weighted and trained

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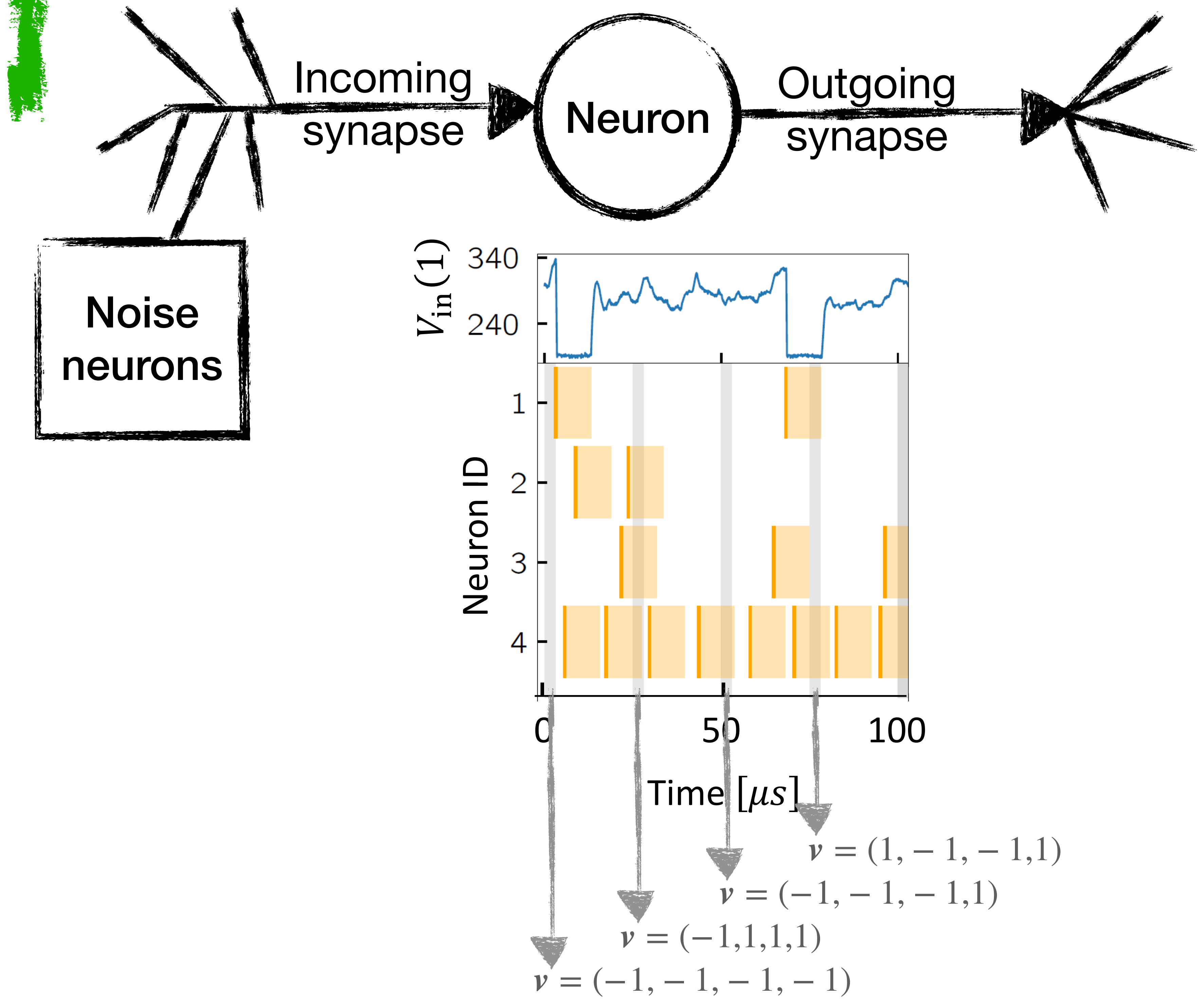
Sampling with Spiking Neurons

[M. Petrovici et al., BMC Neurosci **16** (2015)]

[SC et al., SciPost Physics **12** (2022)]



- Two neuron states: refractory or not
- Add noise neurons for stochasticity
- Read neuron states as samples

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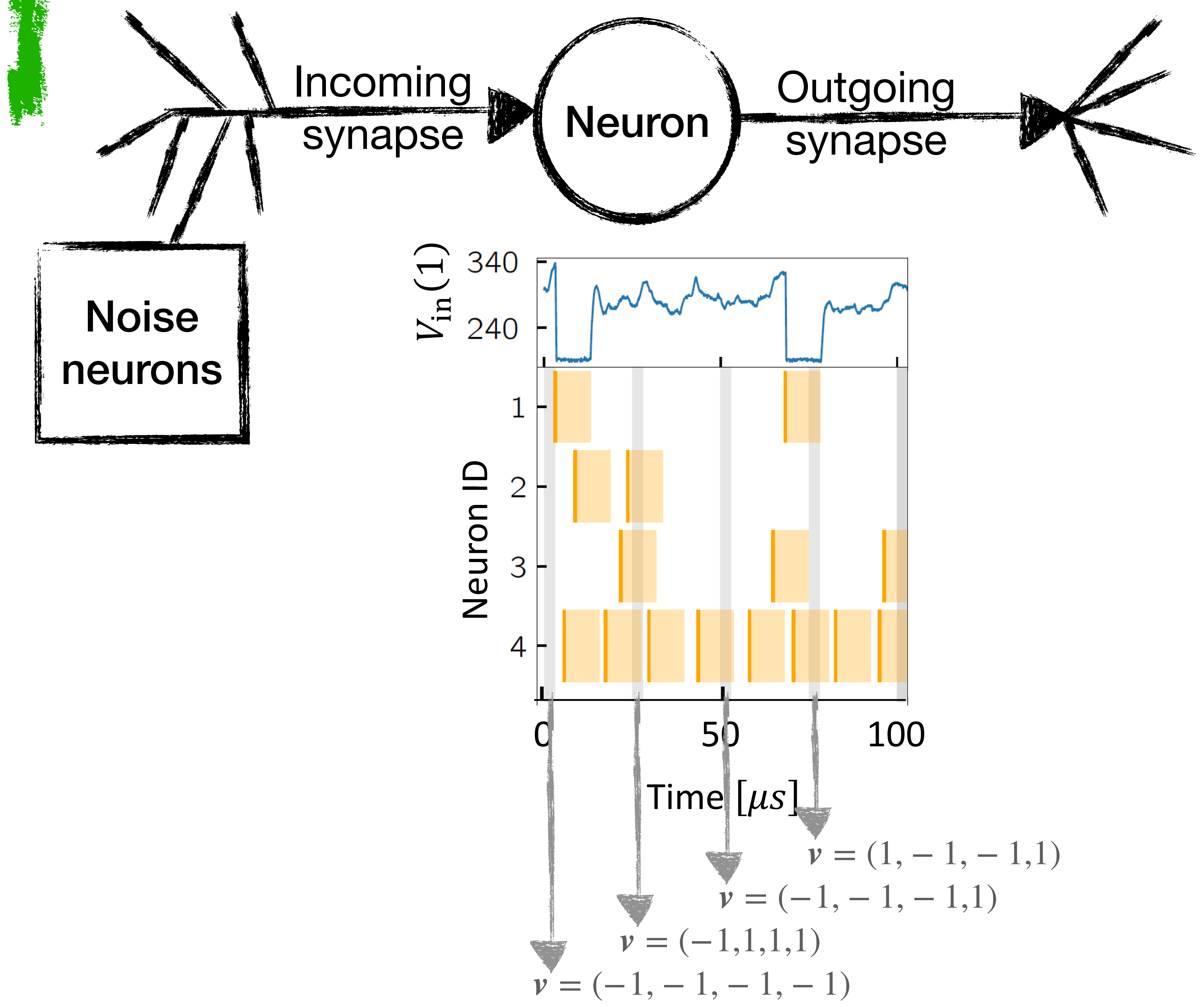

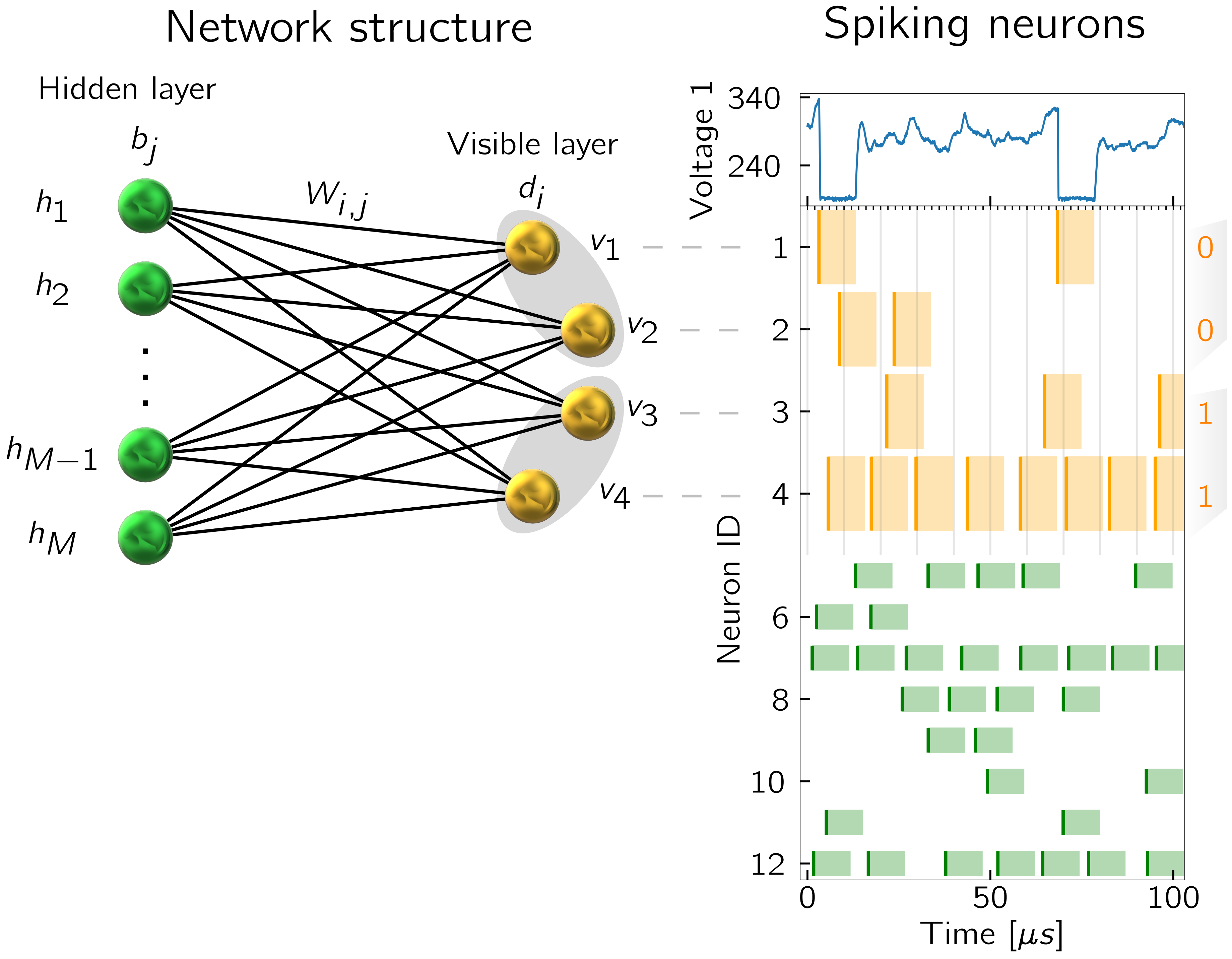

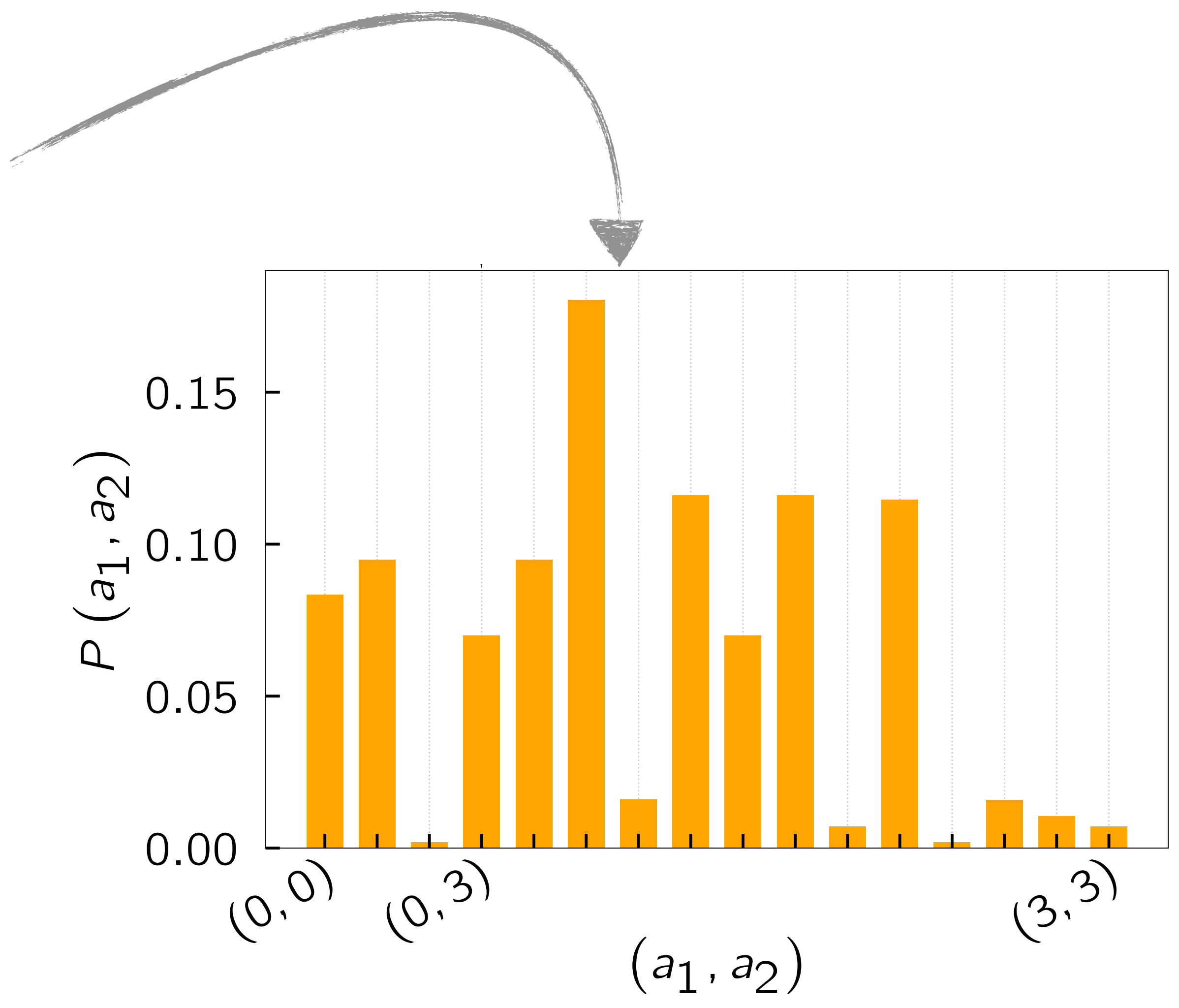

- Two neuron states: refractory or not
- Add noise neurons for stochasticity
- Read neuron states as samples

- Neuron configurations approximate Boltzmann distribution
  - Simulate an RBM

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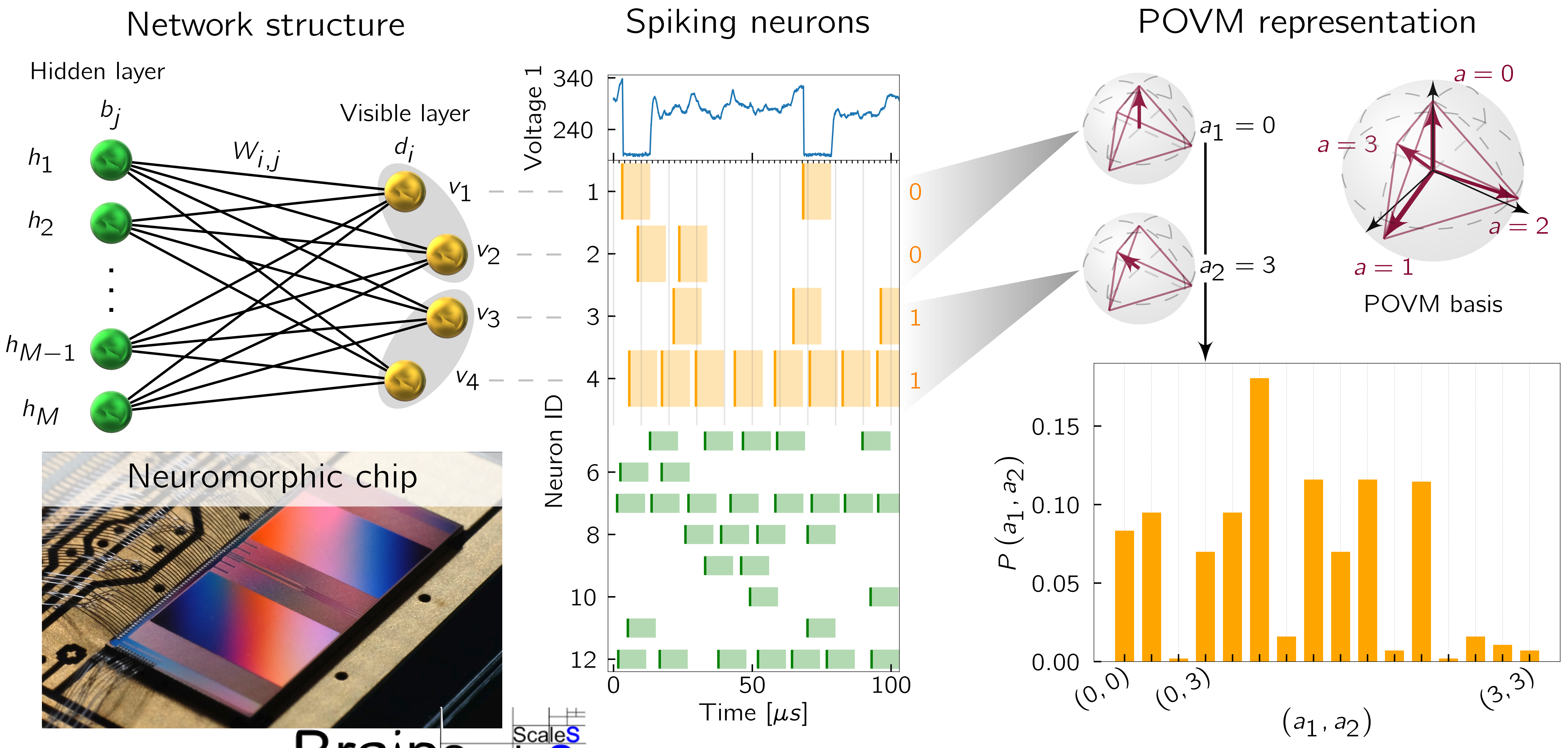
Bell State Reconstruction

[SC et al., SciPost Physics **12** (2022)]

- Train BrainScaleS-2 chip to reconstruct Bell state
- Sample every $2\mu s$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right]$$



BrainScaleS

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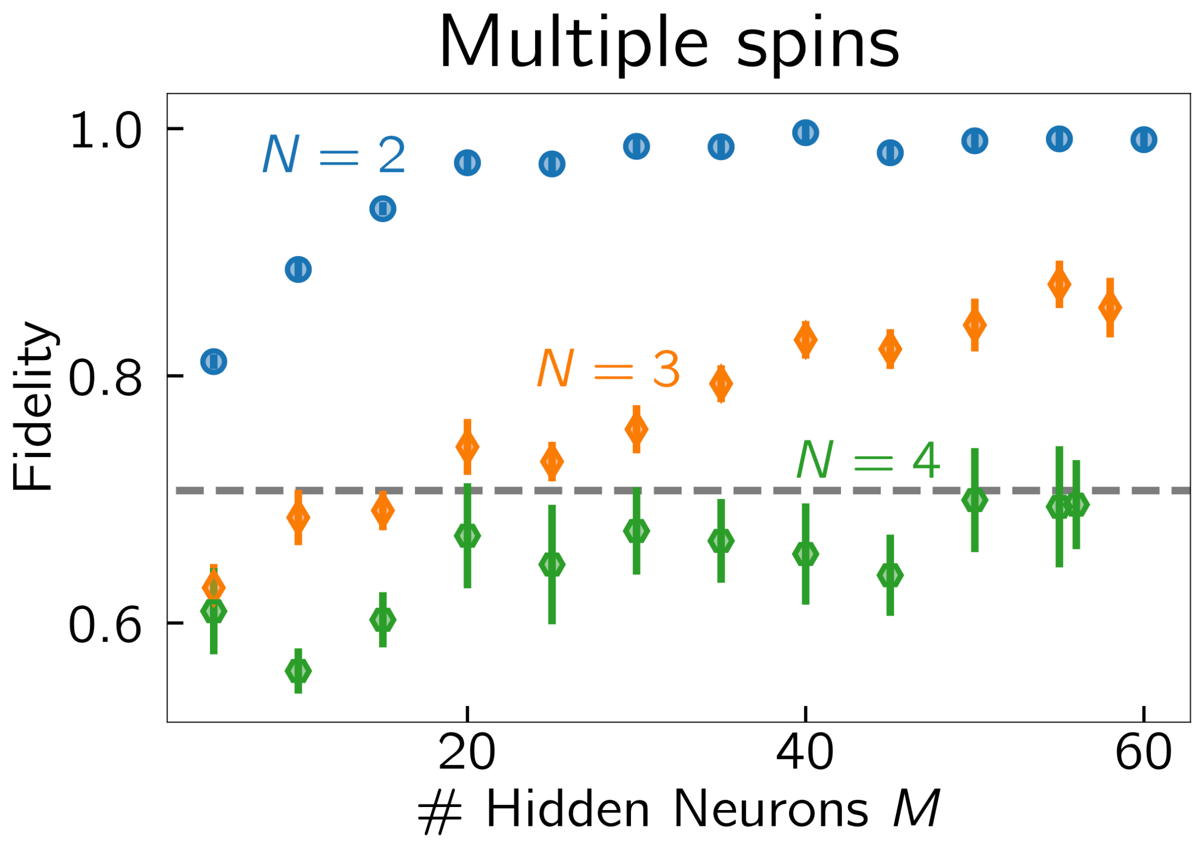
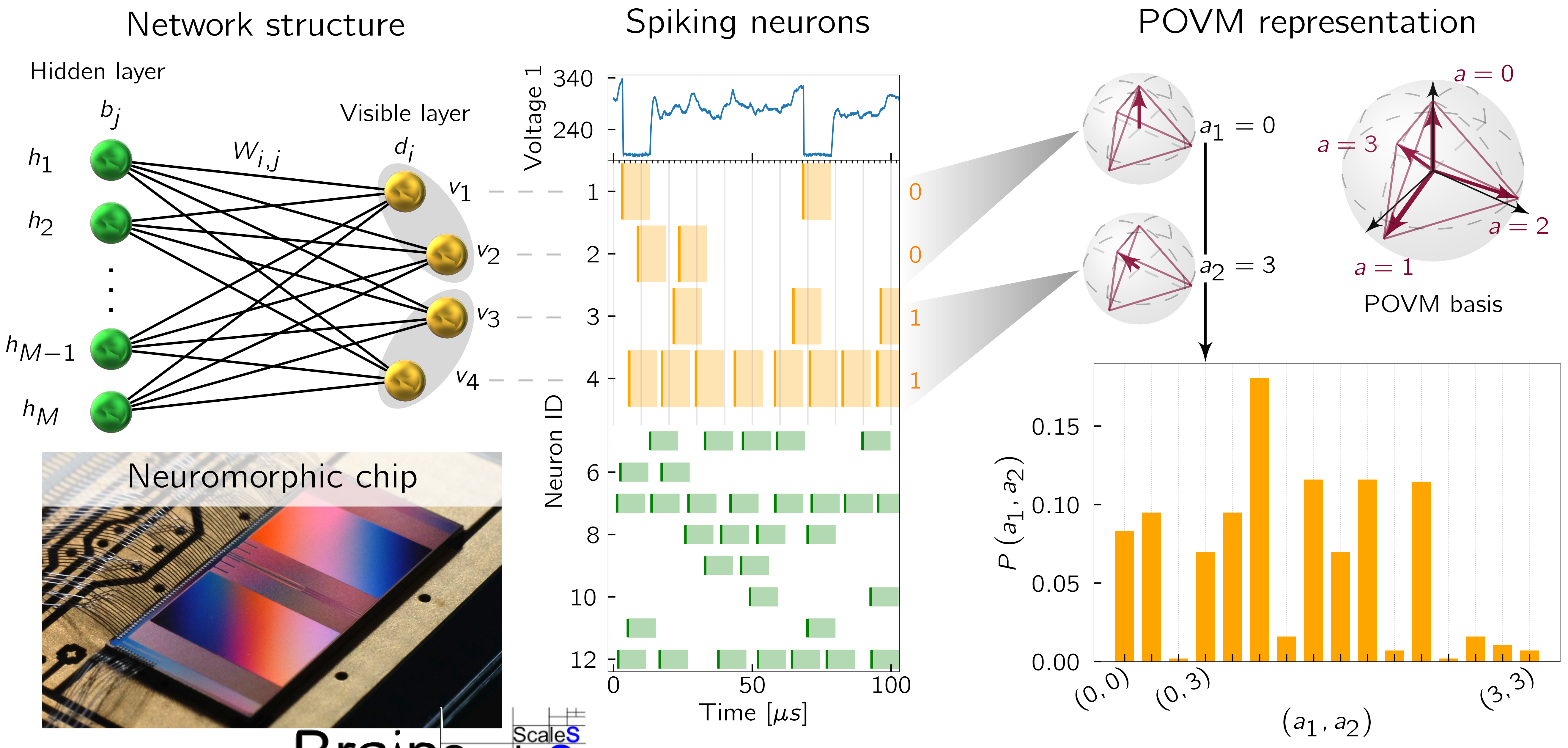
- Sample every $2\mu s$
- High fidelities $\mathcal{F}(\rho_T, \rho_R) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho_T}\rho_R\sqrt{\rho_T}}\right]$
- Limited for larger system sizes





BrainScaleS

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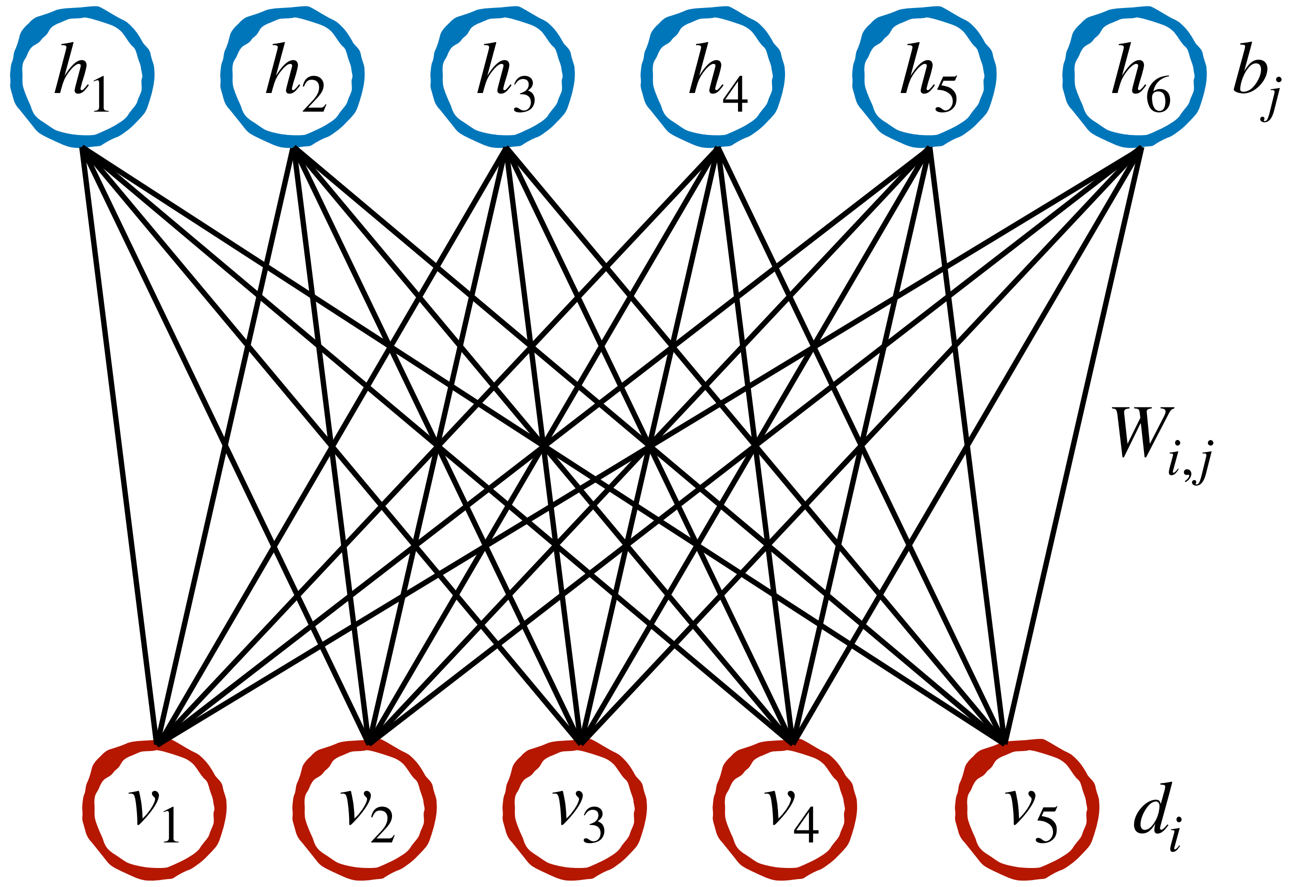
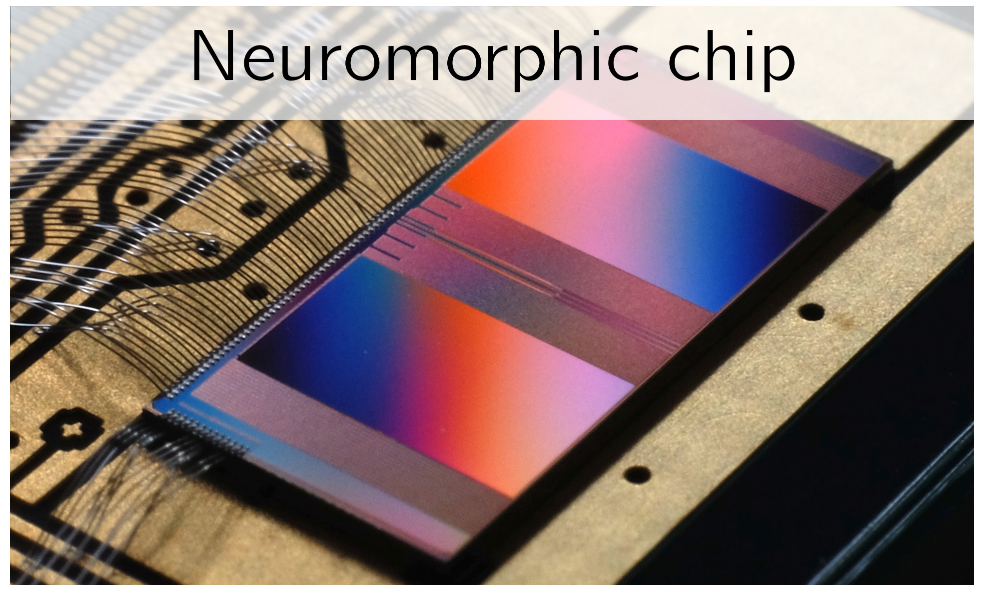
Summary

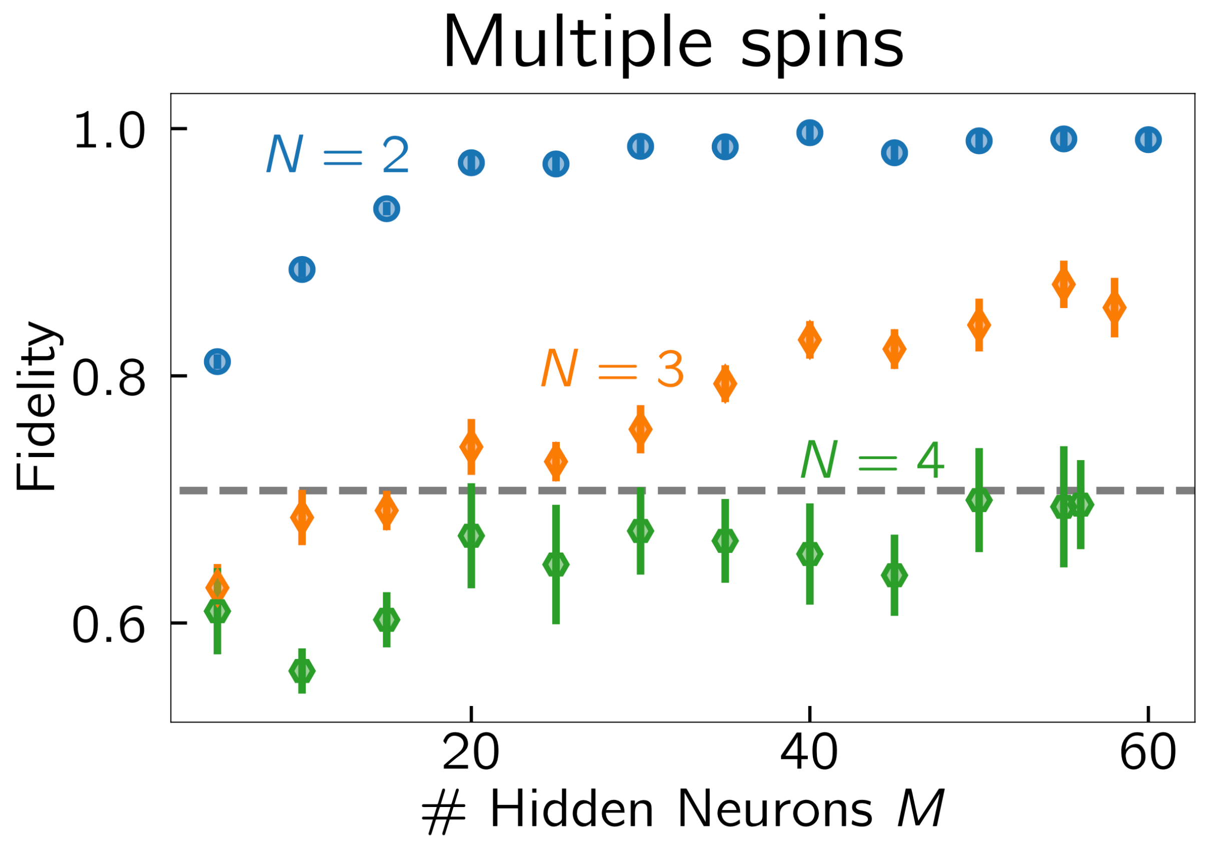
- Quantum state reconstruction
  - Based on measurement data



- Spiking neuromorphic hardware
  - Accelerate sampling
  - Requires many neurons

- Develop more suitable approach





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