Real-Time Distribution System State Estimation with Asynchronous Measurements

Guido Cavraro, E. Dall'Anese, A. Bernstein, J. Comden

Power Systems Engineering Center, NREL, Usa.

Dept. of Electrical, Computer, and Energy Engineering, CU Boulder, USA.

State Estimation for Distribution Network Management

Power systems are deeply changing

- Distribution networks (DNs) host Distributed Energy Resources (DERs)
- Great potential performance improvement
- DNs will need to be managed in real-time
- state estimation plays a fundamental role

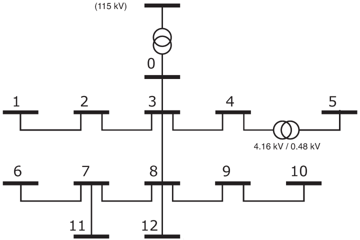
- Classical approaches produce state estimates at the scale of several minutes
- DNs are populated by a number of heterogeneous sensors
- Data fusion of asynchronized measurements
- Process measurements in real-time for faster estimation

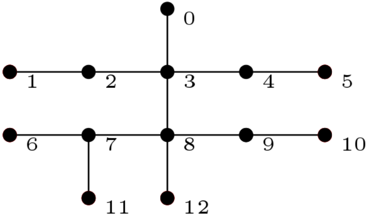
Distribution Network Model

- A single phase DN can be modeled as a graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$
- $\mathcal{N} = \{0, \dots, N\}$, $\mathcal{L} = \{(m, n) : m, n \in \mathcal{V}\}$ collect buses and edges

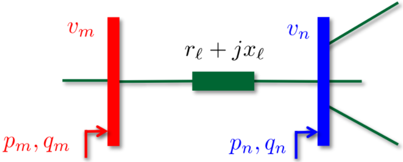
Nodes

- The substation behaves as an ideal voltage generator (Slack bus) with $U_0 = 1$
- p_n, q_n are the active power and the reactive power of node n
- v_n, θ_n are the voltage magnitude and angle of node n
- Collect all these quantities in vectors $\mathbf{p}, \mathbf{q}, \mathbf{v}, \boldsymbol{\theta}$





Distribution Network Model



$$\mathbf{a}_\ell = \begin{bmatrix} \mathbf{0} \\ +1 \\ \mathbf{0} \\ -1 \\ \mathbf{0} \end{bmatrix} \begin{matrix} \\ \longleftarrow \text{ node } m \\ \\ \longleftarrow \text{ node } n \\ \\ \end{matrix}$$

Lines

- $r_\ell + ix_\ell$ impedance of line $\ell = (m, n), \ell \in \mathcal{L}$
- Grid connectivity captured by incidence matrix $\mathbf{A} \in \{0, \pm 1\}^{L \times (N)}$ and by the matrices

$$\mathbf{R} = \left(\sum_{\ell \in \mathcal{L}} \frac{1}{r_\ell} \mathbf{a}_\ell \mathbf{a}_\ell^\top \right)^{-1}, \mathbf{X} = \left(\sum_{\ell \in \mathcal{L}} \frac{1}{x_\ell} \mathbf{a}_\ell \mathbf{a}_\ell^\top \right)^{-1}$$

- **R** and **X** are related with the inverse of the bus admittance matrix

Distribution Network Model

Approximated power flow equations

- Define $\tilde{\mathbf{v}} = \mathbf{v} - \mathbf{1}$
- Voltages can be approximated as

$$\begin{bmatrix} \tilde{\mathbf{v}} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{X} \\ \mathbf{X} & -\mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix}$$

It is convenient to write

$$\begin{bmatrix} \tilde{\mathbf{v}} \\ \boldsymbol{\theta} \\ \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{X} \\ \mathbf{X} & -\mathbf{R} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} = \boldsymbol{\Phi} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix}.$$

S. Bolognani, F. Dorfler (2015)
Fast power system analysis via implicit linearization of the power flow manifold

Measurement model

- Distribution Networks host a variety of sensors
- For simplicity, I'll consider only smart meters and PMU

Smart Meters

- Smart meter at bus m measures $\mathbf{y}_m = [p_m \; q_m \; v_m]^\top$
- Let $\mathbf{S}_m^{SM}$ be a selection matrix; $\mathbf{y}_m$ can be written as

$$\mathbf{y}_m = \mathbf{S}_m^{SM} \boldsymbol{\Phi} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} + \mathbf{n}_m^{SM}$$

PMUs

- PMUs at bus m measures $\mathbf{y}_m = [p_m \; q_m \; v_m \; \theta_m]^\top$
- Let $\mathbf{S}_m^{PMU}$ be a selection matrix; $\mathbf{y}_m$ can be written as

$$\mathbf{y}_m = \mathbf{S}_m^{PMU} \boldsymbol{\Phi} \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \end{bmatrix} + \mathbf{n}_m^{PMU}$$

Dynamic Distribution Network State Estimation

- The state is $\mathbf{x} = [\mathbf{p}^\top \quad \mathbf{q}^\top]^\top$
- Different sensors have different sampling and reporting rates
- We want to process measurements as they come in
- The system at every time t is not observable
- the measurements from the sensors reporting at time t are collected in the vector $\mathbf{y}(t)$,

$$\mathbf{y}(t) = \mathbf{S}(t)\boldsymbol{\Phi}\mathbf{x}(t) + \mathbf{n}(t)$$

- the state estimate at time t is denoted as $\hat{\mathbf{x}}(t)$

Dynamic Distribution Network State Estimation

The state estimator

The state estimate is computed by solving

$$\hat{\mathbf{x}}(t) = \arg\min_{\mathbf{w}} \|\mathbf{y}(t) - \mathbf{S}(t)\mathbf{\Phi}\mathbf{w}\|^2 + \gamma\|\mathbf{w} - \hat{\mathbf{x}}(t-1)\|^2$$

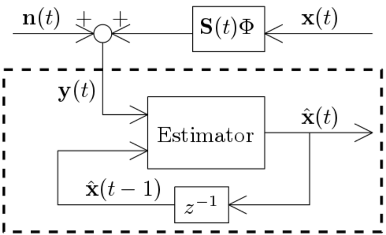
- The first term is a convex LS term
- The second term is a regularization term that makes the problem strictly convex
- the parameter γ is the inertia parameter

We have the closed form expression

$$\hat{\mathbf{x}}(t) = \mathbf{\Lambda}(t)\hat{\mathbf{x}}(t-1) + \frac{1}{\gamma}\mathbf{\Lambda}(t)\mathbf{\Phi}^\top \mathbf{S}(t)^\top \mathbf{y}(t)$$

where $\mathbf{\Lambda}(t)$ is the symmetric and positive definite matrix

$$\mathbf{\Lambda}(t) = \gamma(\mathbf{\Phi}^\top \mathbf{S}(t)^\top \mathbf{S}(t)\mathbf{\Phi} + \gamma\mathbf{I})^{-1}.$$



Dynamic Distribution Network State Estimation

Estimator performance

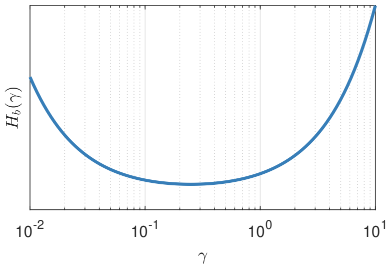
- the maximum state variation is $\Delta_x = \max_t \|\mathbf{x}(t) - \mathbf{x}(t-1)\|$
- the maximum measurement error is $\Delta_n = \max_t \|\mathbf{n}(t)\|$
- τ is a constant such that the DSO gathers measurements from every bus at least once every τ times.

The estimation error $\boldsymbol{\xi}(t) = \hat{\mathbf{x}}(t) - \mathbf{x}(t)$ is asymptotically bounded

$$\limsup_{t\to\infty} \|\boldsymbol{\xi}(t)\| \leq \tau\left(\Delta_x + \frac{c}{\gamma}\Delta_n\right)\left(1 + \frac{\gamma}{\sigma}\right) = H_b(\gamma)$$

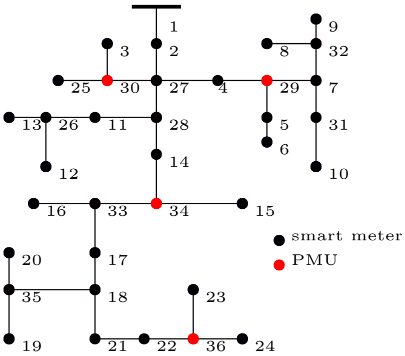
The inertia parameter that minimizes the bound is

$$\gamma^* = \sqrt{\frac{c\sigma\Delta_n}{\Delta_x}}$$



Numerical Tests

- The algorithm has been numerically tested on the 3-phase IEEE 37 bus test feeder
- Red buses are endowed with PMUs
- Black buses are endowed with Smart Meters
- The network states are the power injections
- State estimation performed every minute
- # meas ≤ 10



Simulation Setup

PMUs

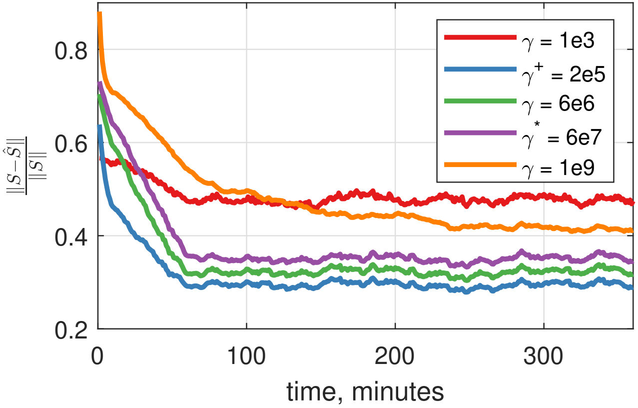
- provide measurements every minute
- introduce a relative error of 0.05 %

Smart Meters

- provide measurements once every hour
- introduce a relative error of 0.5 %

The time after which the system is observable is $\tau = 1$ hour

Effect of the inertia parameter γ



- Small γ makes the estimator very sensitive to noise
- Big γ makes the estimator slow in tracking the state
- γ^* gives almost optimal performance (γ^+)

Comparison with a classical Least Squares Estimator (LSE)

- LSE need a $\#meas \geq \#states$
- When $\#meas < \#states$, pseudo-measurements are used
- The last measurements are used as pseudo-measurements

LSE

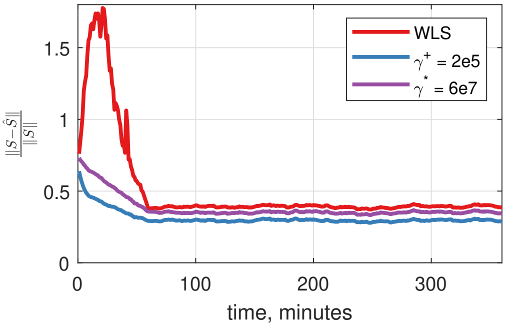
The state estimate is computed by solving

$$\hat{\mathbf{x}}_{LS}(t) = \arg\min_{\mathbf{w}} \|\mathbf{y}(t) - \mathbf{S}(t)\mathbf{\Phi}\mathbf{w}\|^2 + \|\mathbf{y}_{PM} - \mathbf{S}_{PM}\mathbf{\Phi}\mathbf{w}\|^2$$

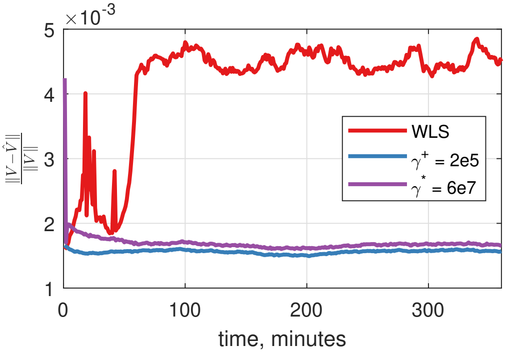
- The first term, built with actual measurement, is convex
- The second term, built with pseudo-measurements, makes the problem strictly convex

Comparison with a classical LSE

Power estimation error



Voltage estimation error



- Voltages are computed solving the Power Flow with the estimated powers
- The initial transitorial behavior is due to the fact that there are not enough pseudo-measurements to make the problem well-posed
- The dynamic estimator outperform the LSE

Conclusions and future developments

We proposed a dynamic state estimator that

- can handle systems with heterogeneous sensors
- is well suited for DNs that are not observable
- outperforms the classic LSE

Future developments include

- adding constraints to the optimization problem
- the use the nonlinear power flow equations

- G. Cavraro, E. Dall'Anese, A. Bernstein. Dynamic Power Network State Estimation with Asynchronous Measurements
  GlobalSip, 2019.
- G. Cavraro, E. Dall'Anese, J. Comden, A. Bernstein. Online State Estimation for Systems with Asynchronous Sensors
  IEEE Transaction on Automatic Control, 2021.

Thanks!

Guido Cavraro

National Renewable Energy Laboratory (NREL)

guido.cavraro@nrel.gov