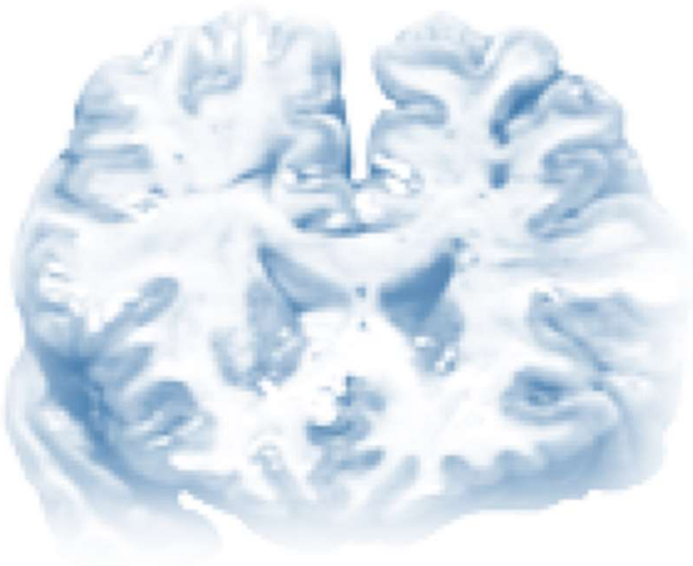
Discovering Hyperelastic Material Models through Sparse Regression with an Application to Human Brain Tissue

AI & Engineering



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Background

- **Human brains** possess highly sophisticated structures and functions owing to the mechanical properties.
- **Mechanical modelling** is a promising tool to understand and predict the behaviour of human brain tissue under extreme conditions such as traumatic injury, shaken baby syndrome and tumor growth.
- **Hyperelastic material models** allow to understand the nonlinear strain-stress relationship of human brain under extremely large elastic deformation up to 700% and various loading modes.
- **Conventional methods** for choosing and calibrating material models are time-consuming, error-prone and call for an automation.
- **Sparse regression** reveals material models from a large candidate model library by excluding irrelevant terms caused by noises, outliers or anormalies in observation.

Hyperelastic model for brain tissue

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Characteristics of hyperelastic material model

- Non-linearly elastic
- Large elastic deformation even to 700 %
- Nearly incompressible
- Suitable for soft flexible materials
- Describable by strain energy density function

Strain energy density function

- Scalar valued function relating strain energy density W of a material to the 3-dimensional deformation gradient $\mathbf{F}$

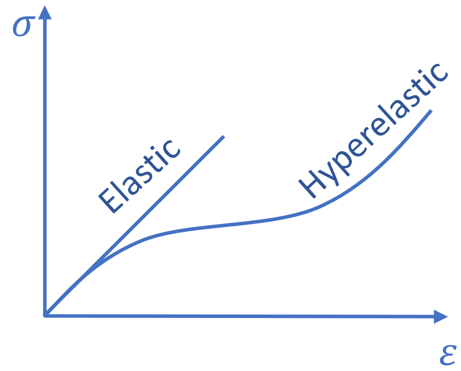
Incompressibility constraint

$$W\left(\mathbf{F}\right) = W\left(\mathbf{F}\right) - p\left(J\left(\mathbf{F}\right) - 1\right)$$

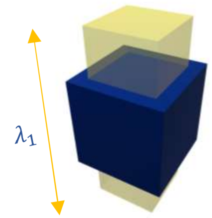
- Stress $(\mathbf{P})$ and torque (τ) under different loads can be derived directly from strain energy density

$$P_{zz} = \left(\frac{\partial W\left(\mathbf{F}\right)}{\partial \mathbf{F}}\right)_{zz}$$

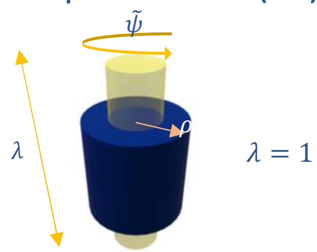
$$\tau = \int_0^1 2\pi\rho^2 \left(\frac{\partial W\left(\mathbf{F}\right)}{\partial \mathbf{F}}\mathbf{F}^T\right)_{\theta z} d\rho$$



Uniaxial tension (UT)



Simple torsion (ST)



Model discovery with sparse regression

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Strain energy density functions can be described with a large feature library

- Linear invariant models
  - Mooney-Rivlin model
  - Logarithmic model
- Non-linear principal stretch models
  - Ogden model

$$W = W_{\text{invariant}} + W_{\text{principal}} = W(\boldsymbol{I}, \boldsymbol{\theta}) + W(\boldsymbol{\lambda}, \boldsymbol{\theta})$$
$$= \boldsymbol{Q}(\boldsymbol{I})^T \boldsymbol{\theta}_{\text{linear}} + W(\boldsymbol{\lambda}, \boldsymbol{\theta}_{\text{non-linear}})$$

- Material parameter vector $\boldsymbol{\theta}$

$$\boldsymbol{\theta}^T = \left[\boldsymbol{\theta}_{\text{linear}}^T, \boldsymbol{\theta}_{\text{non-linear}}^T\right] = \left[\boldsymbol{\theta}_{\text{linear}}^T, \boldsymbol{\mu}^T, \boldsymbol{\alpha}^T\right]$$

Sparse regression

- Refers to reduction of numbers of features for an overdetermined system by turning as many terms of $\boldsymbol{\theta}$ as possible to zeros
- Filters out the most relevant features from the large library
- Reduce the influences of noise, outliers and anomalies of observations

Invariants and principal stretches are dependent on the deformation gradient $\mathbf{F}$.

Mooney-Rivlin

$$\boldsymbol{Q}(\boldsymbol{I}) = \left[(I_1 - 3)^i (I_2 - 3)^{j-i} : \quad j \in \{1, ..., N\}, i \in \{1, ..., j\}\right]^T$$

Logarithmic

$$\boldsymbol{Q}(\boldsymbol{I}) = [\log(I_2/3)]$$

Ogden

$$W(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\alpha}) = \sum_{m=1}^{M} \frac{\mu_m}{\alpha_m^2} \left(\lambda_1^{\alpha_m} + \lambda_2^{\alpha_m} + (\lambda_1 \lambda_2)^{-\alpha_m} - 3\right)$$

Algorithmus of model discovery

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Strain energy density function depending on material parameters

$$W = W\left(\boldsymbol{I},\boldsymbol{\theta}\right) + W\left(\boldsymbol{\lambda},\boldsymbol{\theta}\right) = \boldsymbol{Q}\left(\boldsymbol{I}\right)^T \boldsymbol{\theta}_{\text{linear}} + W\left(\boldsymbol{\lambda},\boldsymbol{\theta}_{\text{non-linear}}\right)$$

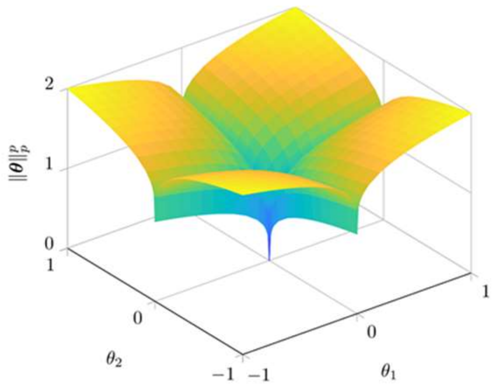
- Stress $P_{zz}\left(\boldsymbol{\theta}\right) = \left(\dfrac{\partial W\left(\boldsymbol{\theta}\right)}{\partial \mathbf{F}}\right)_{zz}$
- Torque $\tau\left(\boldsymbol{\theta}\right) = \displaystyle\int_0^1 2\pi\rho^2 \left(\dfrac{\partial W\left(\boldsymbol{\theta}\right)}{\partial \mathbf{F}}\mathbf{F}^T\right)_{\theta z} d\rho$

Minimization of target function

$$C\left(\boldsymbol{\theta}\right) = \underbrace{\sum_{l=1}^{L}\left(y^{(l)}\left(\boldsymbol{\theta}\right) - \bar{y}^{(l)}\right)^2}_{\text{cost term}} + \overbrace{\lambda_p \left\|\boldsymbol{\theta}\right\|_p^p}^{\text{regularization term}}$$

$$\boldsymbol{\theta} = \arg\min_{\boldsymbol{\theta}}\left(C\left(\boldsymbol{\theta}\right)\right)$$

Increasing the regularization coefficient λ_p leads to more sparse result.



l Load step

$y^{(l)}$ Computed value

$\bar{y}^{(l)}$ Data point

p-norm

$$\left\|\boldsymbol{\theta}\right\|_p^p = \sqrt[p]{|\theta_1|^p + \ldots + |\theta_n|^p}$$

$$p \in (0,1)$$

Result of sparse regression model for brain tissue

Feature library

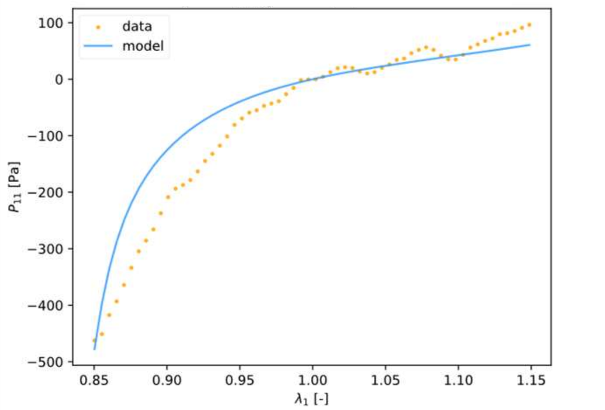
- Linear models: 3-term Mooney-Rivlin model + logarithmic model
- Non-linear model: 3-term Ogden model

$$\boldsymbol{\theta}^T = [\underbrace{\boldsymbol{\theta}_{\text{linear}}^T, \boldsymbol{\mu}^T, \boldsymbol{\alpha}^T}_{Ogden}] = [\underbrace{0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,}_{\boldsymbol{\theta}_{\text{linear}}^T}\ \underbrace{19.3447,\ 0,\ 0,}_{\boldsymbol{\mu}^T}\ \underbrace{-14.2753,\ 0,\ 0}_{\boldsymbol{\alpha}^T}]$$

Result

$$W(\boldsymbol{\lambda}) = \frac{19.3447}{(-14.2753)^2}\left(\lambda_1^{-14.2753} + \lambda_2^{-14.2753} + (\lambda_1\lambda_2)^{14.2753} - 3\right)$$

Uniaxial tension (UT)



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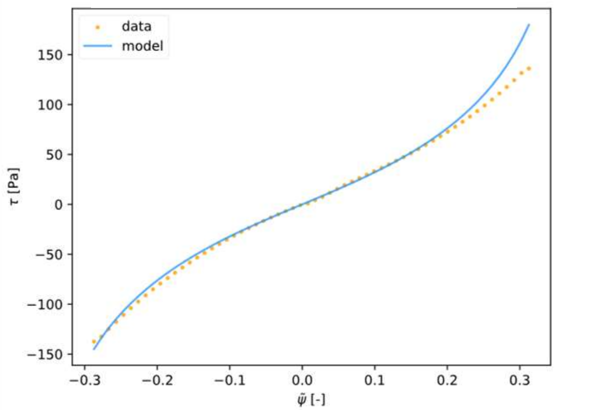

$\lambda_p = 20$

$p = 0.25$

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Thank you for your attention