Deep optimal stopping

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The value of an American-type option is given by

$$\sup_{\tau \in \mathcal{T}} \mathbb{E}[g(\tau, X_\tau)]$$

where

- $(X_t)_{t \in [0,T]}$ is a d-dimensional Markov process on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$
- $g\colon [0,T] \times \mathbb{R}^d \to \mathbb{R}$ is a function
- $\mathcal{T}$ is the set of all X-stopping times τ

The decision to stop at time t must be based on $X_0, \ldots, X_t$!

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Why do we use machine learning?

- Existing methods suffer from the curse of dimensionality and are therefore not feasible for large d!

Deep learning approach

- Learn a candidate optimal stopping time $\hat{\tau}: \Omega \to \{0, T/N, \ldots, T\}$, i.e., for every $t \in \{0, T/N, \ldots, T\}$ train a neural network $f_t: \mathbb{R}^d \to \{0, 1\}$ that decides to stop or not
- $L = \mathbb{E}[g(\hat{\tau}, X_{\hat{\tau}})]$ is a lower bound for $\sup_\tau \mathbb{E}[g(\tau, X_\tau)]$
- Calculate a Monte Carlo estimate $\hat{L} = \frac{1}{M} \sum_{m=1}^{M} L_m$ for L

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Bermudan max-call options

Consider d assets in a multi-dimensional Black–Scholes model

$$X_t^i = s_0^i \exp\left(\left[r - \delta_i - \sigma_i^2/2\right]t + \sigma_i W_t^i\right), \quad i \in \{1, 2, \ldots, d\},$$

where $s_0^i \in (0, \infty)$ (initial values), $r \in \mathbb{R}$ (risk-free interest rate), $\delta_i \in [0, \infty)$ (dividend yields), $\sigma_i \in (0, \infty)$ (volatilities), and $(W_t)_{t \in [0,T]}$ is a d-dimensional Wiener process.

A Bermudan max-call option has time-t payoff $(\max_{1 \leq i \leq d} X_t^i - K)^+$ and can be exercised at one of finitely many times $0 = t_0 < t_1 = \frac{T}{N} < t_2 = \frac{2T}{N} < \cdots < t_N = T$.

Value: $$\sup_{\tau \in \{t_0, t_1, \ldots, T\}} \mathbb{E}\left[e^{-r\tau}\left(\max_{1 \leq i \leq d} X_\tau^i - K\right)^+\right]$$

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Numerical results

Parameters: $s_0^i = 100$, $\sigma_i = 20\%$, $r = 5\%$, $\delta = 10\%$, $K = 100$, $T = 3$, $N = 9$

d	L	t_L	U	t_U	Point est.	95% CI	Binomial	BC 95% CI
2	13.901	27.6	13.903	4.1	13.902	[13.892, 13.932]	13.902	
3	18.694	27.9	18.710	4.1	18.702	[18.677, 18.744]	18.69	
5	26.145	30.2	26.165	4.3	26.155	[26.126, 26.203]		[26.115, 26.164]
10	38.353	32.2	38.357	4.6	38.355	[38.332, 38.401]		
20	51.584	37.4	51.796	5.5	51.690	[51.562, 51.856]		
30	59.512	43.4	59.802	6.1	59.657	[59.490, 59.869]		
50	69.582	55.6	70.008	7.5	69.795	[69.559, 70.101]		
100	83.378	90.5	83.779	11.7	83.579	[83.355, 83.860]		
200	97.422	161.2	97.767	19.3	97.594	[97.398, 97.851]		
500	116.264	450.5	116.645	48.1	116.455	[116.239, 116.733]		

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5	26.145	30.2	26.165	4.3	26.155	[26.126, 26.203]		[26.115, 26.164]
10	38.353	32.2	38.357	4.6	38.355	[38.332, 38.401]		
20	51.584	37.4	51.796	5.5	51.690	[51.562, 51.856]		
30	59.512	43.4	59.802	6.1	59.657	[59.490, 59.869]		
50	69.582	55.6	70.008	7.5	69.795	[69.559, 70.101]		
100	83.378	90.5	83.779	11.7	83.579	[83.355, 83.860]		
200	97.422	161.2	97.767	19.3	97.594	[97.398, 97.851]		
500	116.264	450.5	116.645	48.1	116.455	[116.239, 116.733]		

Numerical results

Parameters: $s_0^i = 100$, $\sigma_i = 20\%$, $r = 5\%$, $\delta = 10\%$, $K = 100$, $T = 3$, $N = 9$

d	L	t_L	U	t_U	Point est.	95% CI	Binomial	BC 95% CI
2	13.901	27.6	13.903	4.1	13.902	[13.892, 13.932]	13.902	
3	18.694	27.9	18.710	4.1	18.702	[18.677, 18.744]	18.69	
5	26.145	30.2	26.165	4.3	26.155	[26.126, 26.203]		[26.115, 26.164]
10	38.353	32.2	38.357	4.6	38.355	[38.332, 38.401]		
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500	116.264	450.5	116.645	48.1	116.455	[116.239, 116.733]		

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