Modeling and Prediction of Non-linearizable Phenomena via Dynamics-based Machine Learning

Mattia Cenedese

Collaborators: J. Axås, B. Kaszás, S. Jain, H. Yang,
M. Eriten, B. Bäuerlein, K. Avila and G. Haller

Lausanne, 28.03.2022

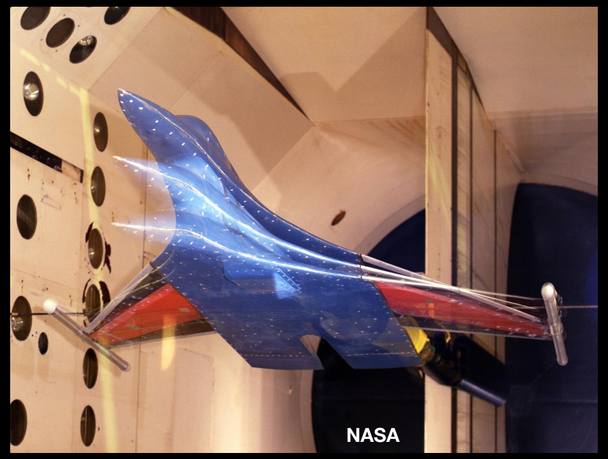
IMES
Institute for Mechanical Systems
Institut für Mechanische Systeme

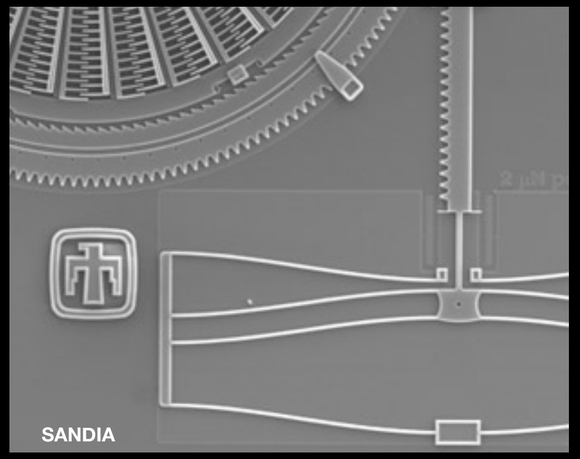
AMLD EPFL

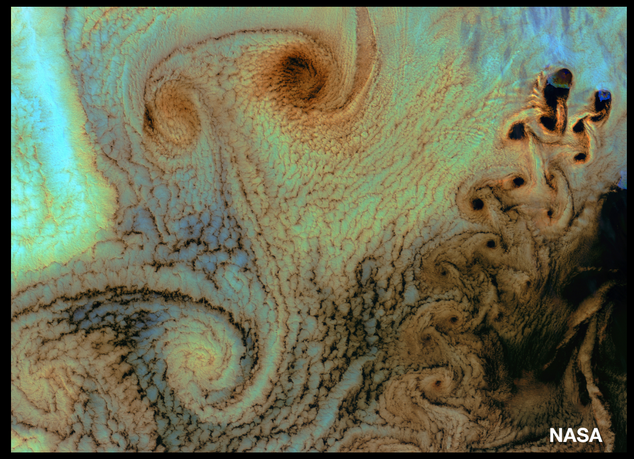
ETH zürich

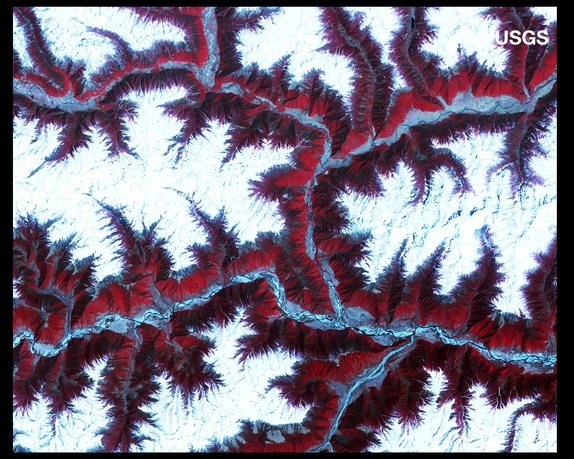
Motivations

- Experimental or numerical analysis of nonlinear high-dimensional dynamical systems

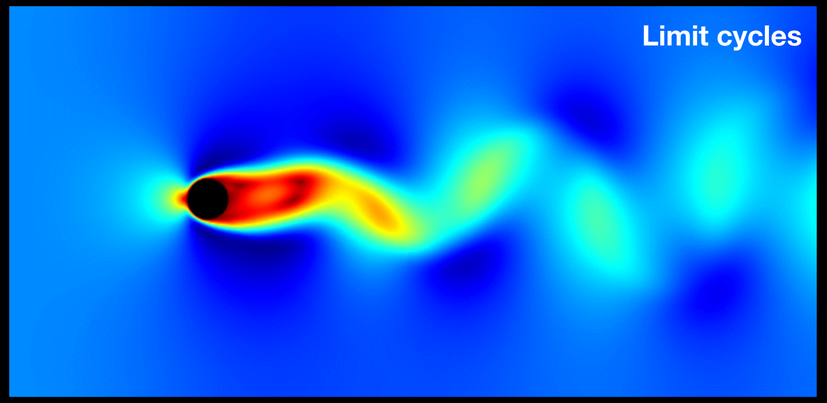


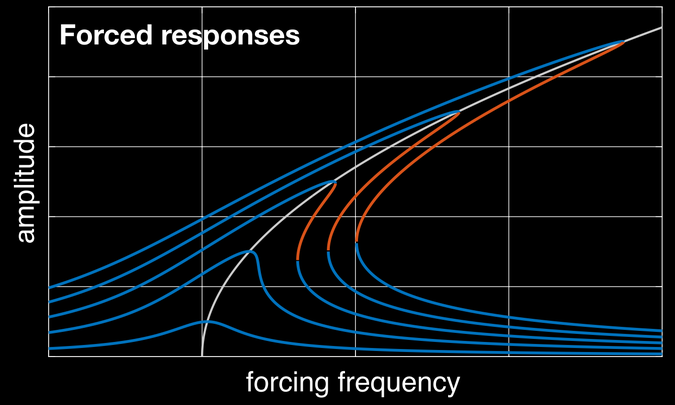


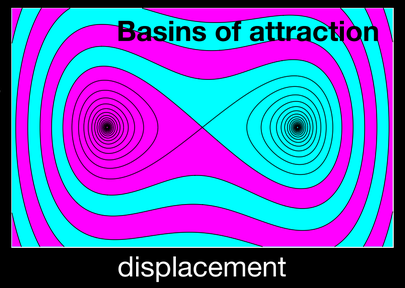


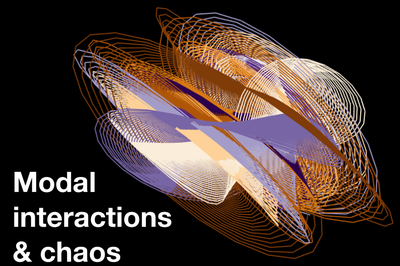


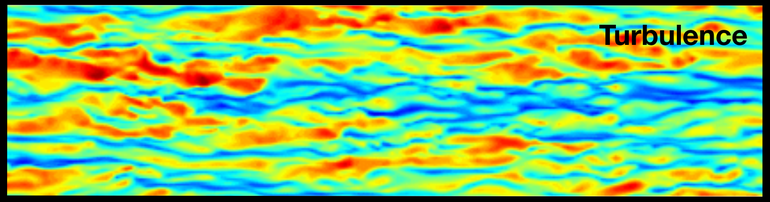
- Develop predictive modeling of non-linearizable dynamics for identification and control











Motivations

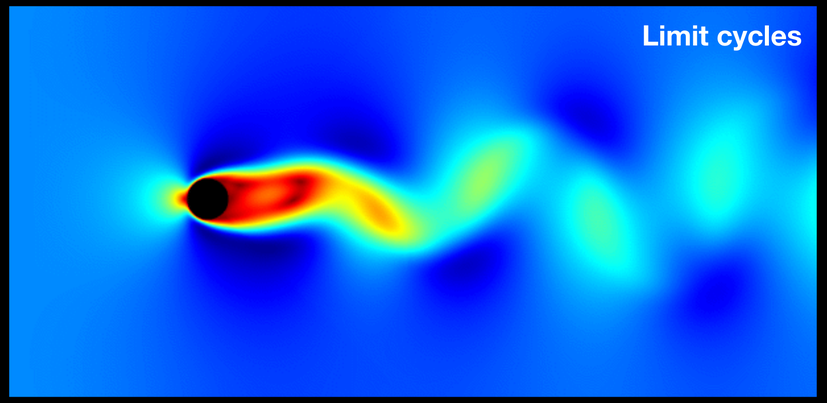
Available methods:

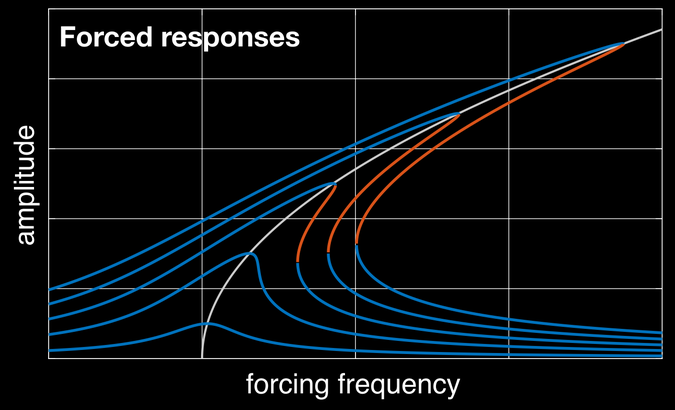
- Dynamic Mode Decomposition
- SINDy
- Neural networks architectures
- Dimensionality reduction with latent coordinates dynamics

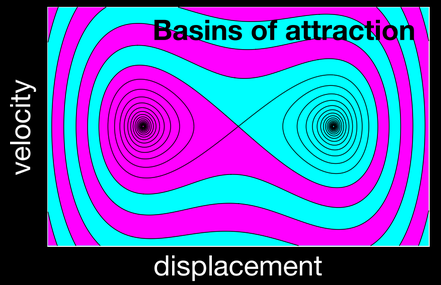
… often cannot:

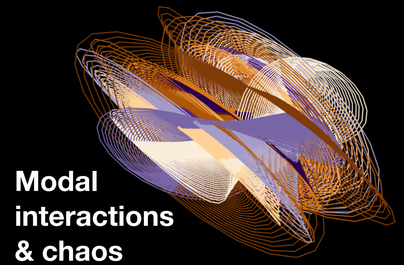
- Return physically meaningful sparse models of practical use
- Make predictions outside their training dynamical regime
- Deal with generic observables of the dynamics

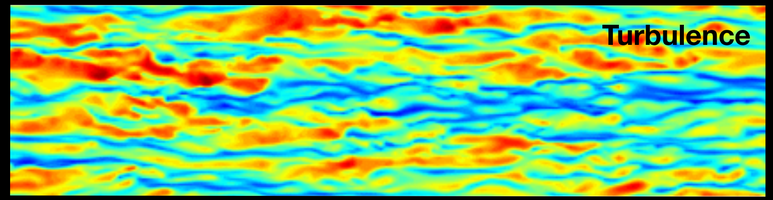
- Develop predictive modeling of non-linearizable dynamics for identification and control







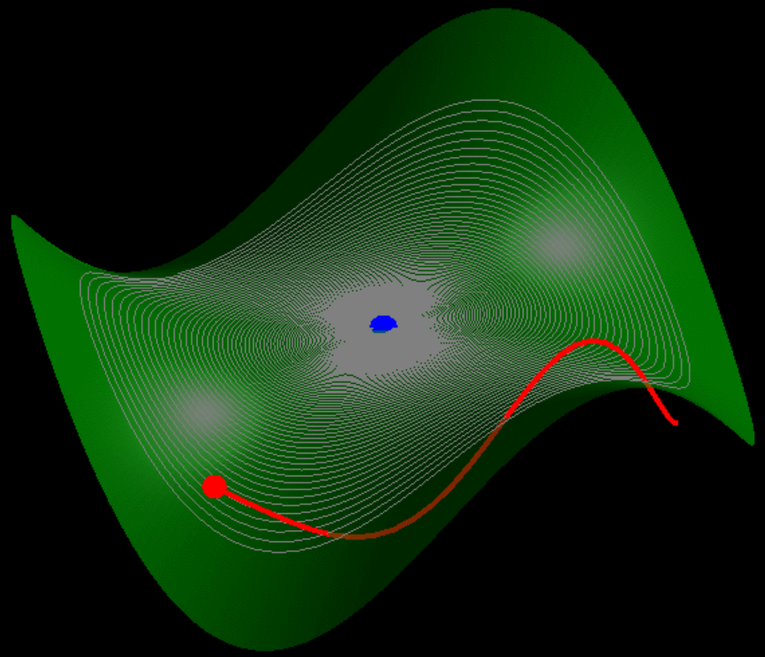


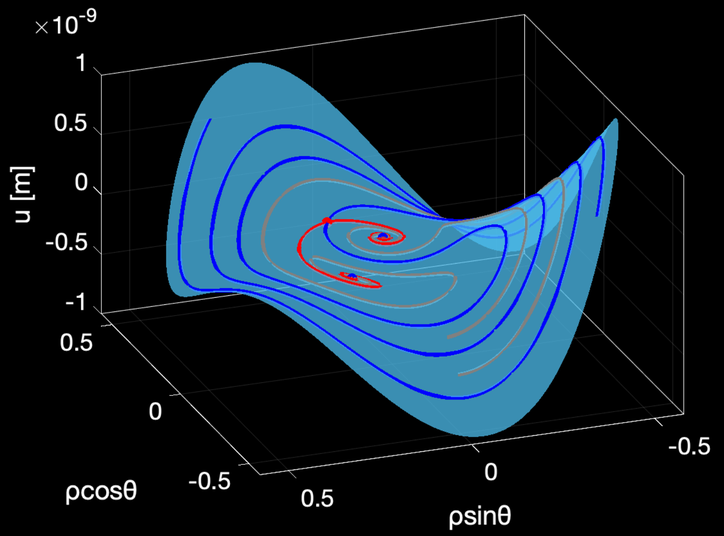


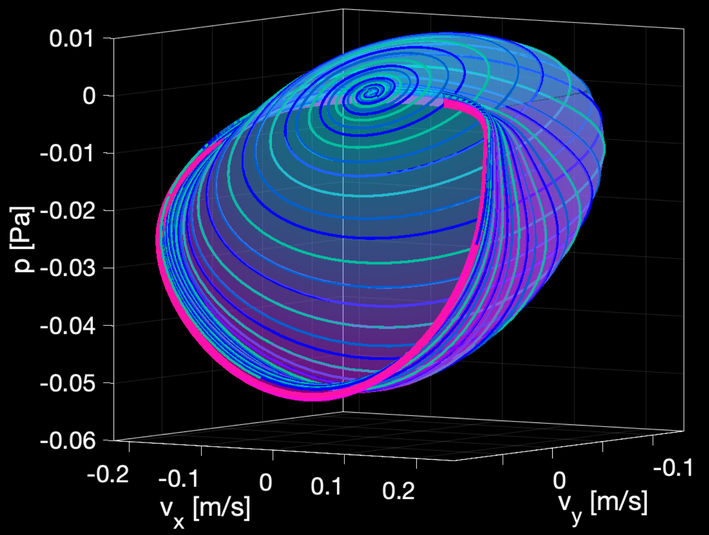
Brunton & Kutz (2019); Karniadakis et al. (2021); Champion et al. (2020)

Dynamics-based machine learning

Focus on key structures of the phase space: **spectral submanifolds** (SSM)







Capture nonlinear phenomena,
also in the presence of external forcing

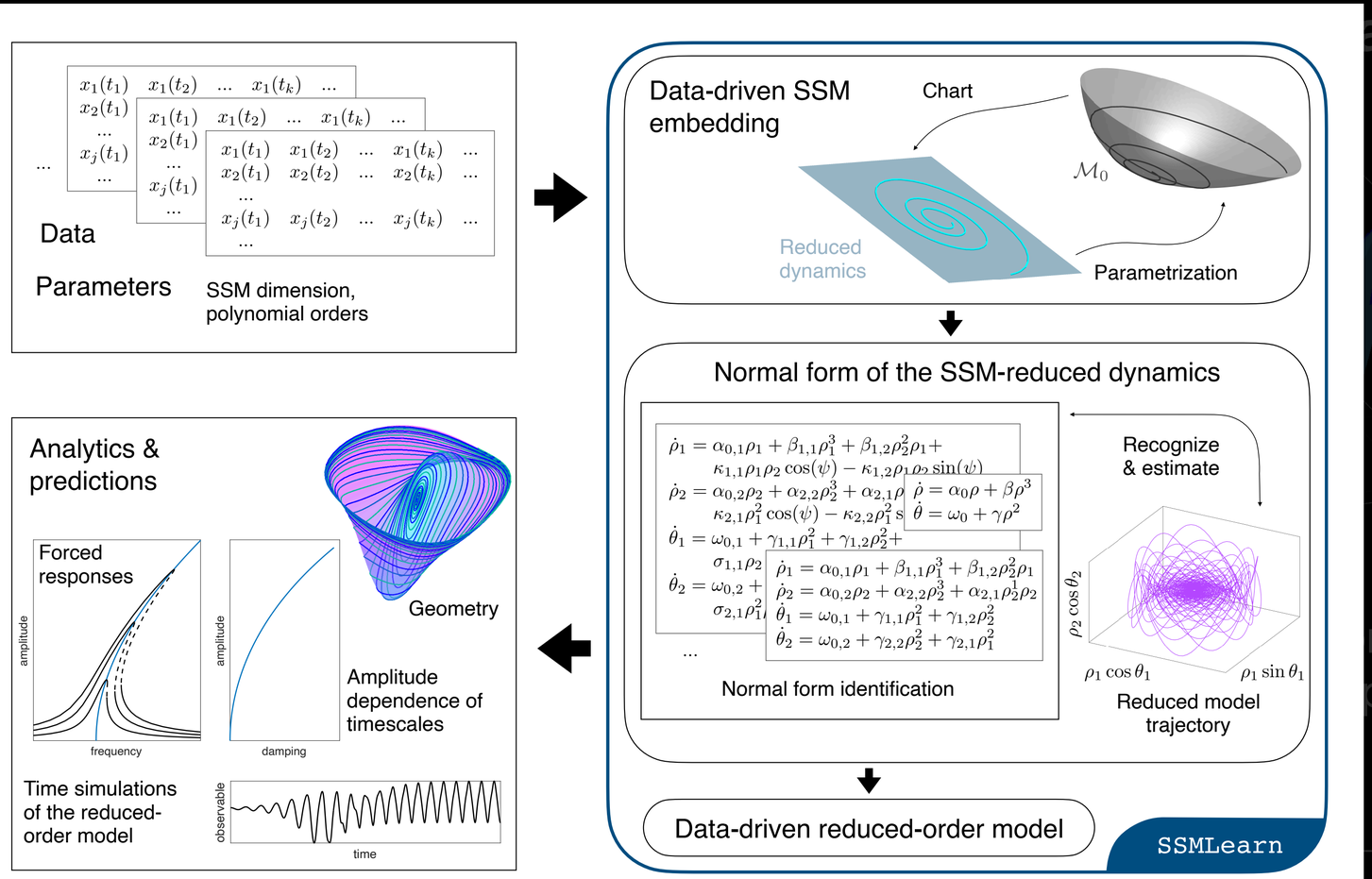
Attracting, robust hypersurfaces
existing in generic embeddings

$$\dot{\rho}_j = \alpha_j(\boldsymbol{\rho}, \boldsymbol{\theta})\rho_j - f_j \sin(\Omega t - \theta_j),$$
$$\rho_j \dot{\theta}_j = \omega_j(\boldsymbol{\rho}, \boldsymbol{\theta})\rho_j + f_j \cos(\Omega t - \theta_j),$$

Their dynamics is described
by simple normal forms

Cenedese, Axås, Bäuerlein, Avila & Haller, *Nat. Commun.* 13 (2022) 872

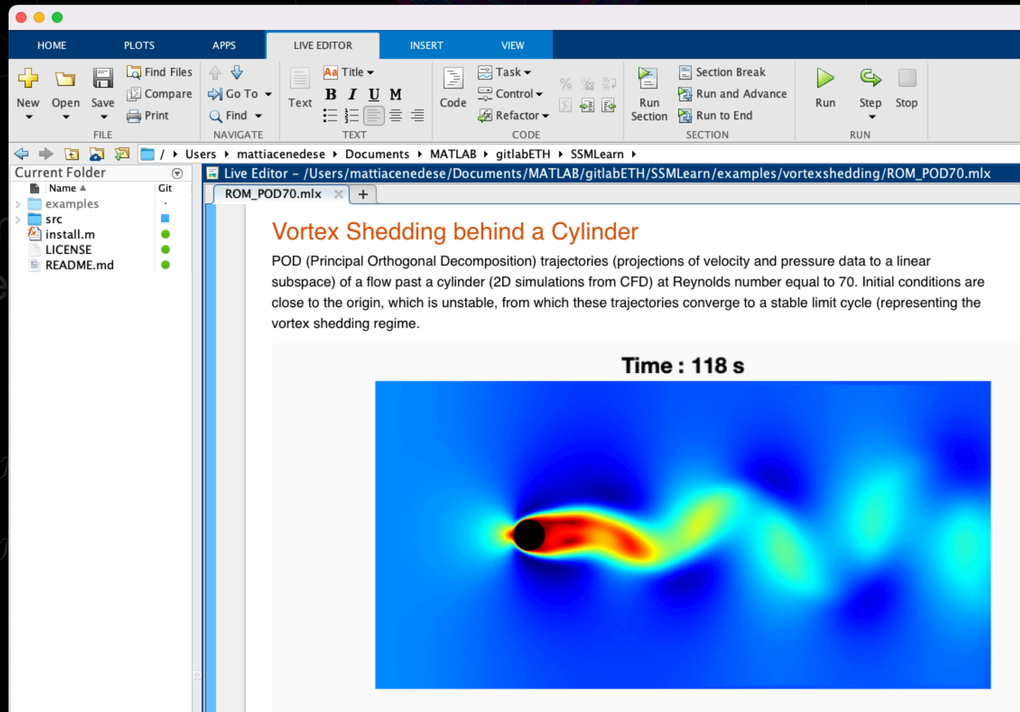
Dynamics-based machine learning



SSMLearn

`github.com/mattiacenedese/SSMLearn`

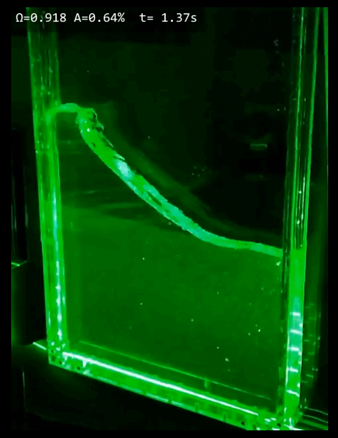
open-source package with guided examples

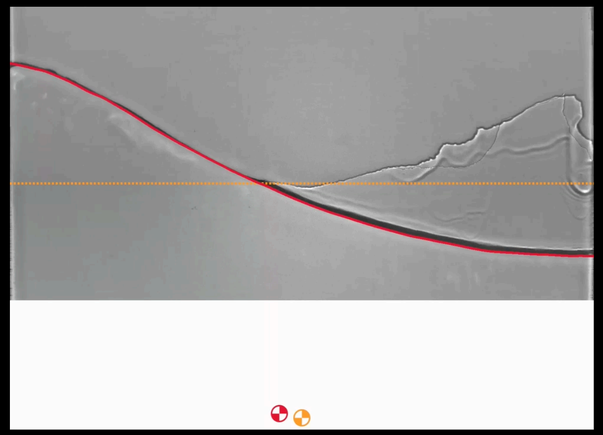


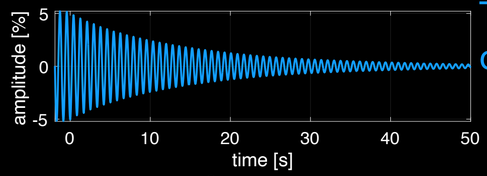
Cenedese, Axås, Bäuerlein, Avila & Haller, *Nat. Commun.* 13 (2022) 872

Results from experimental data

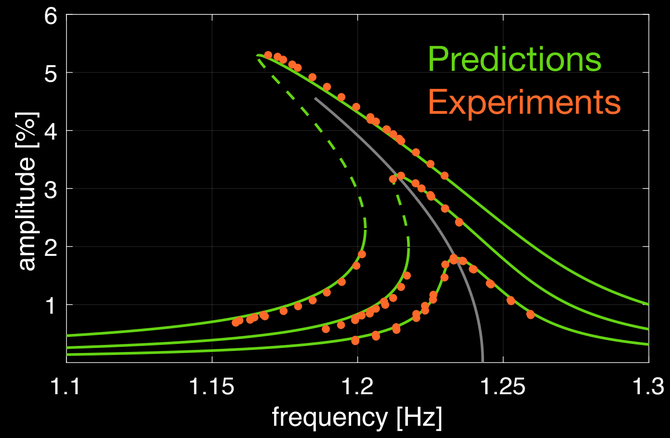
- Prediction of forced responses in liquid sloshing

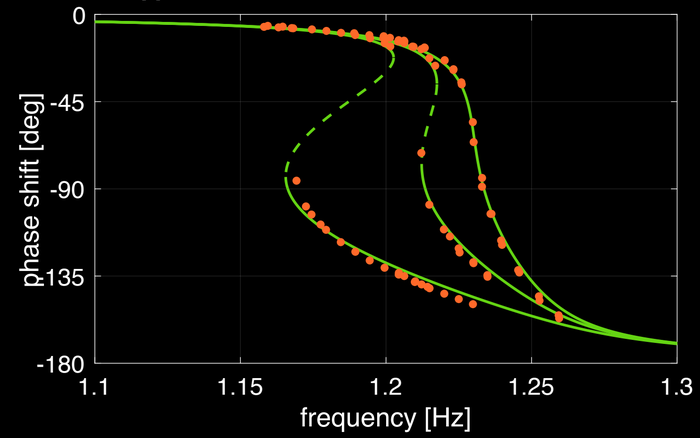




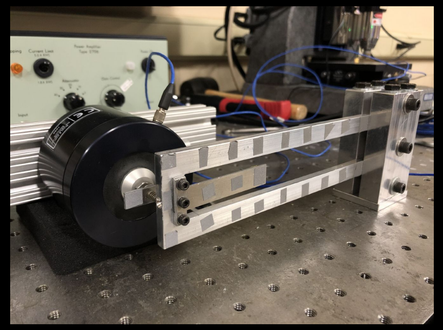


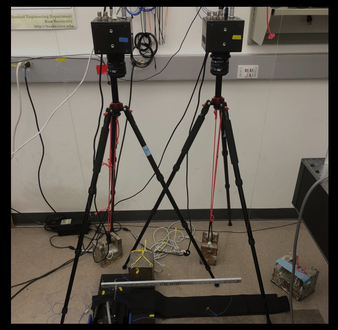
Training data

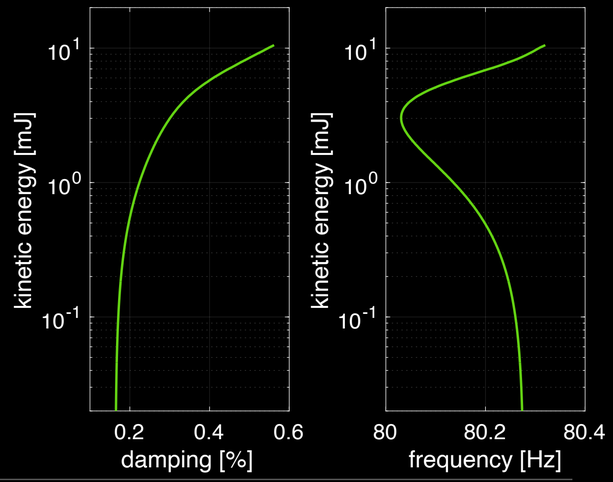


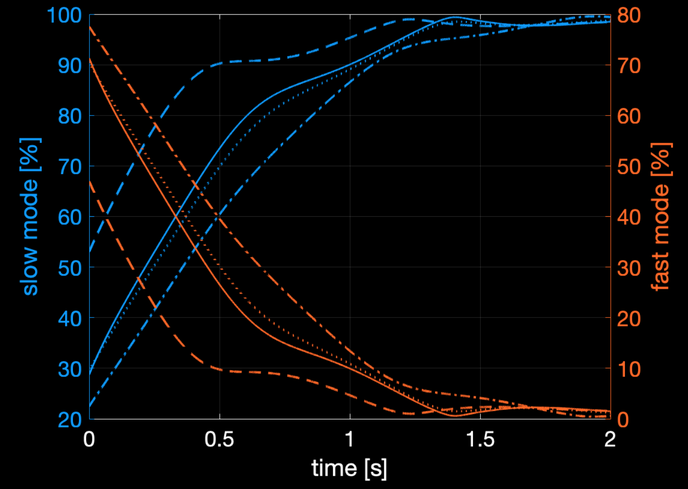


- System identification in nonlinear structures





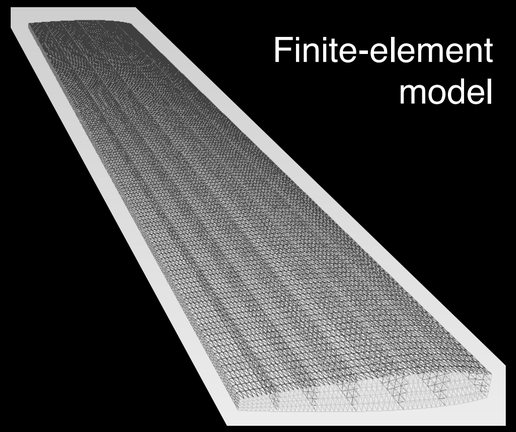


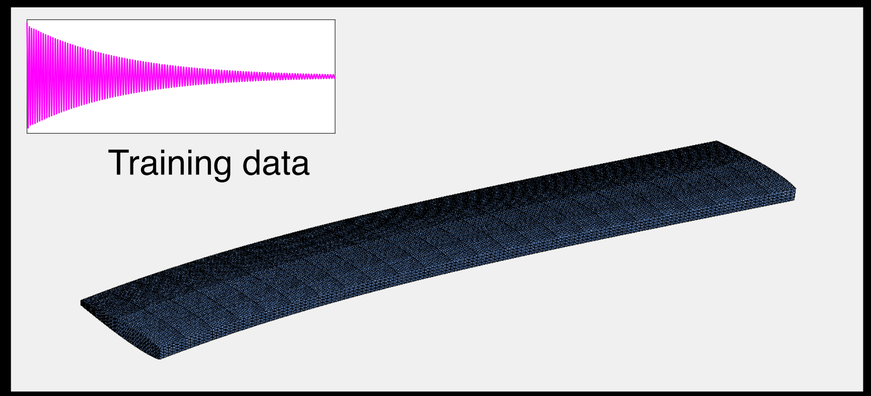


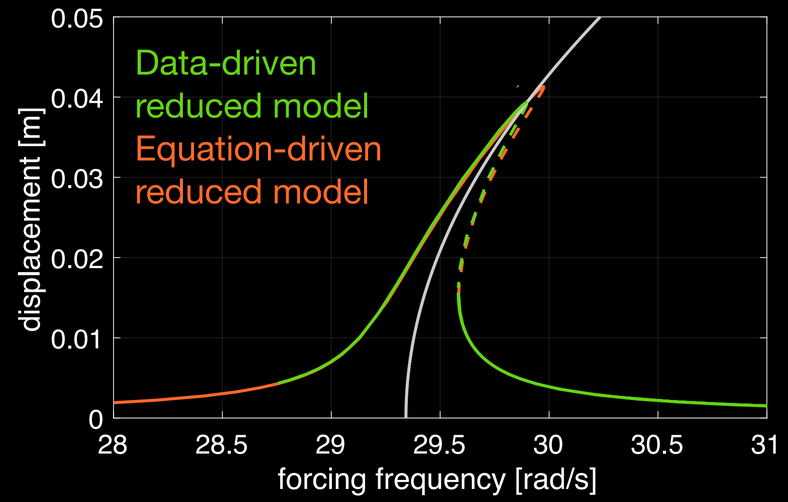
Cenedese, Axås, Bäuerlein, Avila & Haller, *Nat. Commun.* 13 (2022) 872

Results from high dimensional simulation data

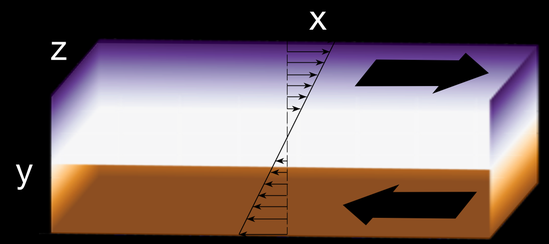
- Model reduction of an airplane wing with 133920 dofs





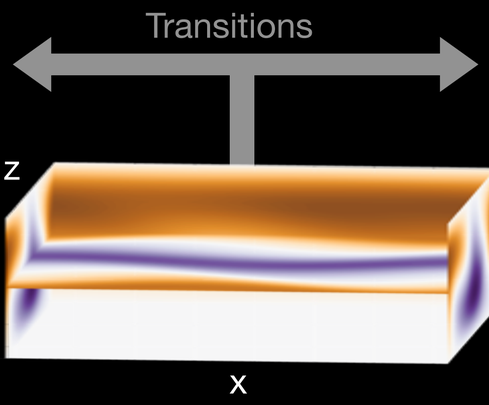


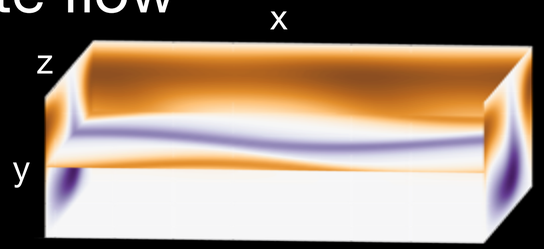
- Prediction of tipping points in Couette flow



Base state

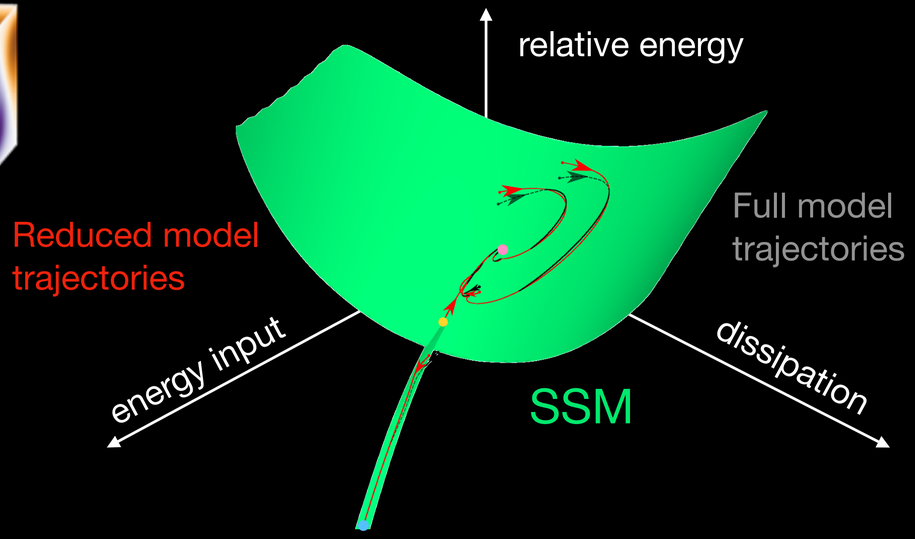
Streamwise velocity
visualizations



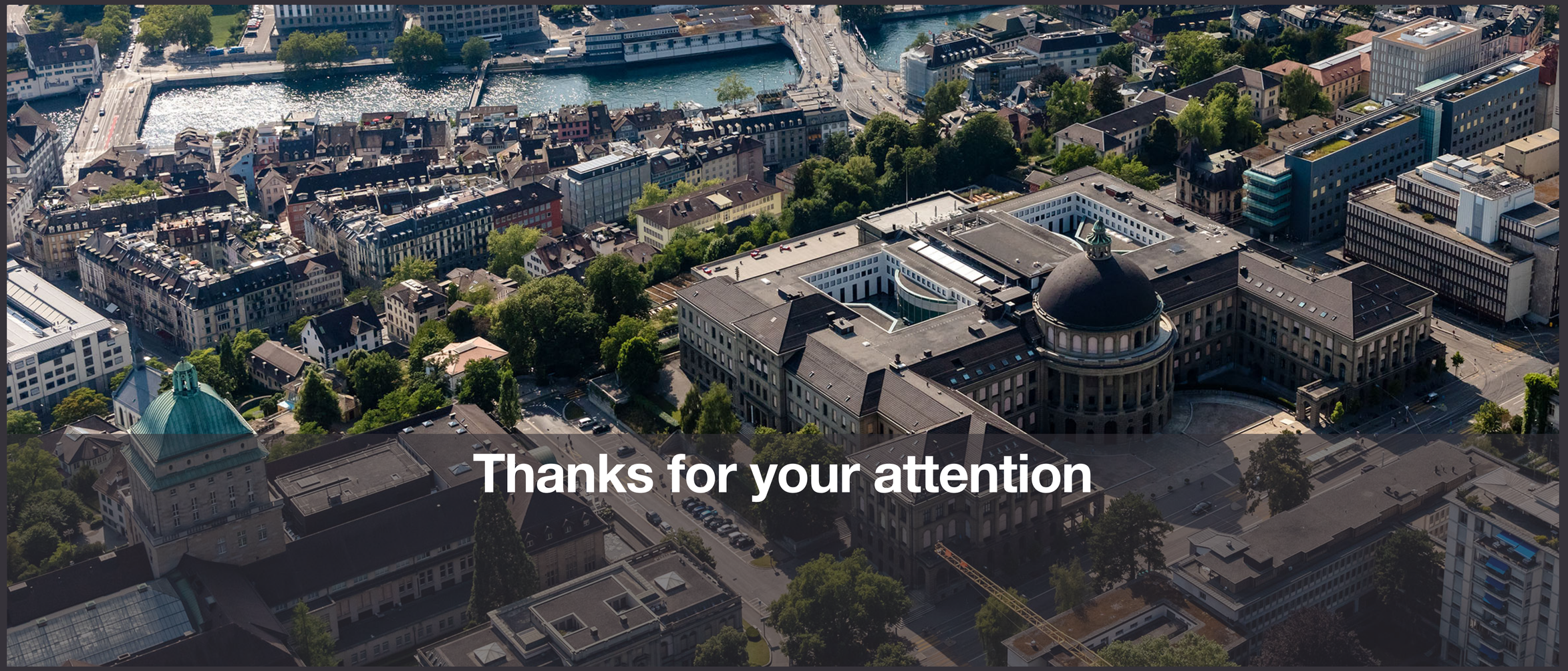


Stable upper state

Unstable lower state



Haller, Jain & **Cenedese**, to apper in *SIAM News;* Kaszás, **Cenedese** & Haller, arXiV (2022)



Check out the codes on `github.com/mattiacenedese/SSMLearn`

IMES
Institute for Mechanical Systems
Institut für Mechanische Systeme

AMLD EPFL

ETH zürich